DOWNLOAD PDF BOOLEAN ALGEBRA AND ITS APPLICATIONS

Chapter 1 : boolean algebra | Download eBook pdf, epub, tuebl, mobi

The first chapter presents the algebra of sets from an intuitive point of view, followed by a formal presentation in chapter two of Boolean algebra as an abstract algebraic system, with no reference to applications.

The laws Complementation 1 and 2, together with the monotone laws, suffice for this purpose and can therefore be taken as one possible complete set of laws or axiomatization of Boolean algebra. Every law of Boolean algebra follows logically from these axioms. Furthermore, Boolean algebras can then be defined as the models of these axioms as treated in the section thereon. To clarify, writing down further laws of Boolean algebra cannot give rise to any new consequences of these axioms, nor can it rule out any model of them. In contrast, in a list of some but not all of the same laws, there could have been Boolean laws that did not follow from those on the list, and moreover there would have been models of the listed laws that were not Boolean algebras. This axiomatization is by no means the only one, or even necessarily the most natural given that we did not pay attention to whether some of the axioms followed from others but simply chose to stop when we noticed we had enough laws, treated further in the section on axiomatizations. Or the intermediate notion of axiom can be sidestepped altogether by defining a Boolean law directly as any tautology, understood as an equation that holds for all values of its variables over 0 and 1. All these definitions of Boolean algebra can be shown to be equivalent. Duality principle[edit] Principle: There is nothing magical about the choice of symbols for the values of Boolean algebra. But suppose we rename 0 and 1 to 1 and 0 respectively. Then it would still be Boolean algebra, and moreover operating on the same values. But if in addition to interchanging the names of the values we also interchange the names of the two binary operations, now there is no trace of what we have done. The end product is completely indistinguishable from what we started with. When values and operations can be paired up in a way that leaves everything important unchanged when all pairs are switched simultaneously, we call the members of each pair dual to each other. The Duality Principle, also called De Morgan duality , asserts that Boolean algebra is unchanged when all dual pairs are interchanged. One change we did not need to make as part of this interchange was to complement. We say that complement is a self-dual operation. The identity or do-nothing operation x copy the input to the output is also self-dual. There is no self-dual binary operation that depends on both its arguments. A composition of self-dual operations is a self-dual operation. The principle of duality can be explained from a group theory perspective by the fact that there are exactly four functions that are one-to-one mappings automorphisms of the set of Boolean polynomials back to itself: These four functions form a group under function composition , isomorphic to the Klein four-group , acting on the set of Boolean polynomials. Walter Gottschalk remarked that consequently a more appropriate name for the phenomenon would be the principle or square of quaternality. There is one region for each variable, all circular in the examples here. The interior and exterior of region x corresponds respectively to the values 1 true and 0 false for variable x. The shading indicates the value of the operation for each combination of regions, with dark denoting 1 and light 0 some authors use the opposite convention. While we have not shown the Venn diagrams for the constants 0 and 1, they are trivial, being respectively a white box and a dark box, neither one containing a circle. However we could put a circle for x in those boxes, in which case each would denote a function of one argument, x, which returns the same value independently of x, called a constant function. As far as their outputs are concerned, constants and constant functions are indistinguishable; the difference is that a constant takes no arguments, called a zeroary or nullary operation, while a constant function takes one argument, which it ignores, and is a unary operation. Venn diagrams are helpful in visualizing laws. The result is the same as if we shaded that region which is both outside the x circle and outside the y circle, i. Digital logic gates[edit.

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Chapter 2 : Boolean Algebra and its Application to Problem Solving and Logic Circuits

This introduction to Boolean algebra begins with an intuitive approach to set theory and an axiomatic account of the fundamentals of Boolean algebra, proceeding to concise accounts of applications to symbolic logic, switching circuits, relay circuits, binary arithmetic, and probability theory.

One can easily derive many elementary laws from these axioms, keeping in mind this example for motivation. The two-element BA shows the direct connection with elementary logic. The two members, 0 and 1, correspond to falsity and truth respectively. An important elementary result is that an equation holds in all BAs if and only if it holds in the two-element BA. This makes the ring into a BA. These two processes are inverses of one another, and show that the theory of Boolean algebras and of rings with identity in which every element is idempotent are definitionally equivalent. This puts the theory of BAs into a standard object of research in algebra. A BA is atomic if every nonzero element of the BA is above an atom. Finite BAs are atomic, but so are many infinite BAs. We can generalize this: These do not exist for all sets in all Boolean algebras; if they do always exist, the Boolean algebra is said to be complete. The elementary algebraic theory Several algebraic constructions have obvious definitions and simple properties for BAs: Some other standard algebraic constructions are more peculiar to BAs. Although not immediately obvious, this is the same as the ring-theoretic concept. There is a dual notion of a filter with no counterpart in rings in general. This establishes the basic Stone representation theorem, and clarifies the origin of BAs as concrete algebras of sets. This establishes a one-one correspondence between the class of BAs and the class of such spaces. As a consequence, used very much in the theory of BAs, many topological theorems and concepts have consequences for BAs. This is an algebraic expression of the disjunctive normal form theorem of sentential logic. Another general algebraic notion which applies to Boolean algebras is the notion of a free algebra. This can be concretely constructed for BAs. There are many special classes of Boolean algebra which are important both for the intrinsic theory of BAs and for applications: Atomic BAs, already mentioned above. Atomless BAs, which are defined to be BAs without any atoms. For example, any infinite free BA is atomless. Complete BAs, defined above. These are specially important in the foundations of set theory. These BAs are useful in the study of Lindenbaum-Tarski algebras. Every countable BA is isomorphic to an interval algebra, and thus a countable BA can be described by indicating an ordered set such that it is isomorphic to the corresponding interval algebra. These are BAs which are not only atomic, but are such that each subalgebra and homomorphic image is atomic. Structure theory and cardinal functions on Boolean algebras Much of the deeper theory of Boolean algebras, telling about their structure and classification, can be formulated in terms of certain functions defined for all Boolean algebras, with infinite cardinals as values. Every infinite complete BA has an independent subset of the same size as the algebra. Every interval algebra has countable independence. A superatomic algebra does not even have an infinite independent subset. Every tree algebra can be embedded in an interval algebra. A BA with only the identity automorphism is called rigid. There exist rigid complete BAs, also rigid interval algebras and rigid tree algebras. More recently, many cardinal functions of min-max type have been studied. For example, small independence is the smallest size of an infinite maximal independent set; and small cellularity is the smallest size of an infinite partition of unity. Decidability and undecidability questions A basic result of Tarski is that the elementary theory of Boolean algebras is decidable. Even the theory of Boolean algebras with a distinguished ideal is decidable. On the other hand, the theory of a Boolean algebra with a distinguished subalgebra is undecidable. Both the decidability results and undecidablity results extend in various ways to Boolean algebras in extensions of first-order logic. Lindenbaum-Tarski algebras A very important construction, which carries over to many logics and many algebras other than Boolean algebras, is the construction of a Boolean algebra associated with the sentences in some logic. The simplest case is sentential logic. Here there are sentence symbols, and common connectives building up longer sentences from them: Any BA is isomorphic to one of this form. One can do something similar for a first-order theory. However, one of the most important uses of these classical Lindenbaum-Tarski algebras is to describe them for important theories usually decidable theories. For countable languages this can be done by describing their isomorphic interval algebras. Generally this gives a thorough knowledge of the theory. Theory Isomorphic to interval algebra on 1.

Chapter 3 : Boolean Algebra - body, used, form, system, Applications

Boolean Algebra and Its Applications by J. Eldon Whitesitt This introduction to Boolean algebra explores the subject on a level accessible even to those with a modest background in mathematics. The first chapter presents the algebra of sets from an intuitive point of view, followed by a formal presentation in chapter two of Boolean algebra as.

Boolean Algebra Boolean algebra Boolean algebra is a form of mathematics developed by English mathematician George Boole â€" Boole created a system by which certain logical statements can be expressed in mathematical terms. The consequences of those statements can then be discovered by performing mathematical operations on the symbols. As a simple example, consider the following two statements: The rules of Boolean algebra can be used to find out the consequences of various combinations of these two propositions, P and Q. In general, the two statements can be combined in one of two ways: I will be home today OR I will be home tomorrow. I will be home today AND I will be home tomorrow. Now the question that can be asked is what conclusions can one draw if P and Q are either true T or false F. For example, what conclusion can be drawn if P and Q are both true? In that case, the combination P or Q is also true. That is, if the statement "I will be home today" P is true, and the statement "I will be home tomorrow" Q is also true, then the combined statement, "I will be home today OR I will be home tomorrow" P or Q must also be true. By comparison, suppose that P is true and Q is false. That is, the statement "I will be home today" P is true, but the statement "I will be home tomorrow" Q is false. Most problems in Boolean algebra are far more complicated than this simple example. Over time, mathematicians have developed sophisticated mathematical techniques for analyzing very complex logical statements. Applications Two things about Boolean algebra make it a very important form of mathematics for practical applications. First, statements expressed in everyday language such as "I will be home today" can be converted into mathematical expressions, such as letters and numbers. Second, those symbols generally have only one of two values. The statements above P and Q , for example, are either true or false. That means they can be expressed in a binary system: Binary mathematics is the number system most often used with computers. Computer systems consist of magnetic cores that can be switched on or switched off. The numbers 0 and 1 are used to represent the two possible states of a magnetic core. Boolean statements can be represented, then, by the numbers 0 and 1 and also by electrical systems that are either on or off. As a result, when engineers design circuitry for personal computers, pocket calculators, compact disc players, cellular telephones, and a host of other electronic products, they apply the principles of Boolean algebra.

Chapter 4 : Boolean Algebra and Its Applications | Bookshare

1 Boolean Algebra And Its Applications Introduction Let Î© be a set consisting of two elements denoted by the symbols 0 and 1, i.e. Î©= {0,1}.

Binary numbers should be introduced in grade school, since they are easy and give insight into the usual decimal number system. Each 1 or 0 is called a bit, short for binary digit. Such operations are facilitated by coding the binary number, which is grouping the digits and giving symbols to the groups. Hexadecimal coding is the favorite method, in which the bits are grouped in fours, and the numbers , A-F are assigned to the groups corresponding to their values. The only disadvantage of hexadecimal is the necessity for introducing the symbols A-F for If we group by threes instead, we need only the digits This octal coding was once frequently seen, but has now fallen into desuetude. In C, we would write 0x To convert to decimal, you take the first digit and multiply by 16, then add the second and multiply by 16 again, and so on, until reaching the final digit, which is simply added. A similar rule, but with division by 16, converts from decimal to hexadecimal. There are also ways to express decimal fractions as hexadecimal fractions and vice-versa, but this is seldom required. So, normally we handle binary numbers as hexadecimal for the usual operations of arithmetic. Suppose we consider 8-bit numbers bytes. Computers handle negative numbers this way, since only the usual binary arithmetic is required. Boolean Algebra In general, algebra represents quantities as symbols which are manipulated according to certain rules. Geometry represented such quantities as the lengths of lines, which were manipulated by the processes of geometry, which is usually a much more difficult matter. Algebra, a word taken from Arabic, is the only major contribution of the Arabs to mathematics, though they appreciated and studied the subject with great intelligence. It was developed in its present form in Italy in the 15th and later centuries, where many new problems were solved, and became an essential part of the new infinitesimal analysis known as the Calculus, and through this the basis of mathematical physics. There is a more restricted definition of an algebra in mathematics: George Boole introduced what has become known as Boolean Algebra as a mathematical representation of propositional logic, where statements could be either true or false, and nothing else. Symbols representing these values could be manipulated according to certain rules to demonstrate the consistency of a chain of reasoning, or to find new results proceeding from the old. We have symbols and two operations on them, but no scalar product and so no field F , plus some additional operations peculiar to quantities that take two values only. Therefore, Boolean algebra is not strictly an algebra, but only one in a general sense. Boolean algebra applies wherever we have quantities that can assume only one of two values, and so has a remarkably wide field of application and a very practical one. It can be applied abstractly to sets of objects, where the binary thing is inclusion or non-inclusion algebra of classes. The calculus of propositions applies to statements that are either true or false, with a variety of connectives, not simply the two of an algebra, but expressible in terms of them. A switch is either open or closed, another binary variable, so switching algebra is an application of concern to us here. Digital signals, which assume only two values, are another application closely related to switching algebra. Boolean algebra is fascinating to study, which is perhaps the principal reason for considering it, but it can also give some help in the specification and design of switching or digital networks. This help is valuable, but often over-rated. Boolean algebra by itself cannot solve practical problems. In the diagram at the right, let the large circles enclose all the points under consideration, a universe or population represented by I or 1. If the answer to "is every point included" is yes, then this is represented by I. The small circle in the left-hand diagram represents those points in a smaller set, denoted by x. The Boolean variable x is 1 when a point is in the smaller set, 0 when it is not. That is, x is a function over the points of the complete set I, that is 1 for points inside the small circle. Exactly the same things are represented by the circle at the right divided into right and left halves. The only difference is the precise set of points represented by x, which is of no importance in the algebra. Therefore, the regions in such diagrams as these, when expressing general algebraic relations, are of arbitrary shape and size. These are called Venn diagrams, and are useful for visualizing Boolean algebra. A special form of the Venn diagram is the Karnaugh K- map. For one variable x, these are the 2-cell figures shown at the right. Each cell represents an elementary area in the Venn diagram. The cells may also be labelled by the values of the variable, 0 or 1, instead of by the convention shown, and this is usually done in digital design. Each cell may contain a 1 or be left empty, which is equivalent to a 0. At this point, the K-maps are only curiosities. In logic or circuits, the similar operation is called OR. A Venn diagram that shows two subsets x and y illustrates what happens for different subsets. Note that x and y refer to the complete circles, not just to the area the letters are in. What are these areas? We can split the Venn diagram into elementary areas, and combine these in various ways, which is equivalent to doing algebra with the symbols, applying the rules of Boolean algebra. It is clear that OR and AND are associative and commutative, like ordinary addition and multiplication. These rules can be proved with Venn diagrams. The proof of the second distributive law is shown at the right. The dotted area can be expressed in two ways, which gives the result. It is mainly the idempotency and the second distributive law that make Boolean algebra seem strange, but it is all quite valid. The term that disappears is called the consensus term. If both y and z are true, then one of the first two terms must also be true. You may have noticed that the rules occur in pairs, in which OR and AND are interchanged, as well as the constants 1 and 0. Such results are called dual, and arise from the fact that only two values are possible. The last step makes the relation not the dual, but the same function, it should be noticed. Karnaugh maps for two variables are shown at the left. This may not be the only way to express it, but it is one way. Of these, the one corresponding to x AND y is shown at the upper left. All can be written down at once from the definitions of the functions. Think of each cell as part of a Venn diagram to write out an expression for the function. This is the simplest example of such areas in a Karnaugh map. The function represented by a 1 in each of the four cells, in which the whole universe is covered, is simply the constant 1. When all the cells are empty, the function is the constant 0. Show that the complement of x XOR y is the same as its dual. K-maps are merely a cunning way to write the truth table for a Boolean function. In a truth table, the inputs are listed, usually in binary number order, and opposite each set of inputs is the value of the function, 1 or 0. A Boolean function is completely specified by its truth table, and the truth table contains a finite number of rows. Digital Design In digital design, the Boolean variables are high H and low L voltage levels on conductors that are interpreted as 1 and 0 positive logic or 0 and 1 negative logic , and the two cases are the duals of each other. A Boolean function is the output of a circuit that depends on the levels of the input conductors. For negative logic, it is an OR gate. Similarly, a circuit that outputs an L only when both inputs are L is called an OR gate LS32 , which it is for positive logic. For negative logic, it is an AND gate. Only two levels are used in digital logic so that errors of interpretation can be reduced to insignificance, trading bandwidth for this essential feature. Computers must operate without digital errors. When a human makes an error, an exceedingly common event, it is only an inconvenience and a delay. An error in a computer system can bring everything crashing down, and is no trivial matter. Fast modems, which make use of multilevel logic, are made possible by automatic error detection and correction. Work this out for yourself, and show that it is true. Two inversion bubbles on the same conductor eat each other, but if they occur at the two ends, they are generally left to show that the logic level is inverted on this line. With these rules, you can modify digital circuits to find a better configuration. For Boolean functions of one or two variables, there is little need for advanced methods, and solutions quickly suggest themselves. One simply has to be familiar with the available integrated circuit packages and their characteristics. Most actual problems are of this type, but they can hardly be considered problems. For four variables, the K-map is specially convenient, since it can be drawn in a simple form, and the problems can be complex enough for the method to be valuable. The case of three variables is intermediate; K-maps are easily drawn and used, but the problems are not very difficult. In every case, however, the principles are the same. For more variables than four or possibly five, computer methods of simplification minimization are useful and rather powerful. However, the profitablility of such analyses is doubtful, and are applicable to problems that are no longer of much importance, since complex logic is no longer constructed from discrete gates. A K-map for a function of four Boolean variables is shown at the right. Note how the values of w, x, y and z are arranged, so that the areas on the Venn diagram that are contiguous are contiguous in the K-map. In going from one cell to a neighboring one, horizontally or vertically, the value of only one variable may change. Hence the order is 00, 01, 11, not the order of increasing binary numbers. Also note that the diagram is continued from the right edge to the left, and from the bottom to the top, since the regions are contiguous.

Chapter 5 : Boolean Algebra | www.nxgvision.com

The project "Applications of Boolean Algebra: Claude Shannon and Circuit Design" is designed for an introductory or intermediate course in discrete or finite mathematics that considers boolean algebra from either a mathematical or computer science perspective.

Boolean Algebra In George Boole â€” , an English mathematician, published one of the works that founded symbolic logic. His combination of ideas from classical logic and algebra resulted in what is called Boolean algebra. Using variables and symbols, Boole designed a language for describing and manipulating logical statements and determining if they are true or not. The variables stand for statements that are either true or false. Although truth tables use T and F for true and false respectively to indicate the state of the sentence, Boolean algebra uses 1 and 0. The relationship between Boolean algebra, set algebra, logic, and binary arithmetic has given Boolean algebra a central role in the development of electronic digital computers. Besides its many applications in the design of computers, it forms the foundation of information theory. Truth Tables Boolean algebra is based on propositions, which are non-ambiguous sentences that can be either true or false. One can combine these propositions in a variety of ways by using the connectives and and or, or one can negate them by preceding them with not. The results of these operations on propositions are dictated by the rules of Boolean algebra. For example, if one says: Therefore the properties of "mittens" and "green" will have to be present in all her "hand-covering" purchases. This will exclude gloves and all non-green mittens. How does this work out using truth tables? Let A represent "mittens," B represent "green. What the tables indicate is that if an item does not possess both the quality of being a mitten and the quality of being green, then it will be discarded. Only those that satisfy both qualities will be selected. On the other hand, if one says: This means that he will have a great assortment of "hand-covering" garments. Let A represent "mittens" and B represent "gloves. What the tables indicate is that an item will be selected if it possesses both qualities of mitten and glove, or possesses only one quality, either glove or mitten. Only those that satisfy neither quality will be discardedâ€"for example, all red socks. One can also say: The tables indicate that if an item is a mitten then its negation, â€"A, represents a non-mittenâ€"for example, a glove or a sock. Computer Design Boolean algebra can be applied to the design and simplification of complex circuits present in computers because computer circuits are two-state devices: This corresponds to the general representation of Boolean algebra with two elements, 0 and 1. To show how this works, take a look at two simple circuits, "and," and "or," which correspond to the first two sets of tables presented earlier. These simple circuits consist of a power sourceâ€"a battery connected by a wire to a destinationâ€"and a lamp with two switches that control the flow of electricity. The position of a switch either allows electricity to flow from the power source to the destination, or stops it. For example, if the switch is up, or open, then electricity does not flow, and this condition is represented by a 0. However, if the switch is down, or closed, the electricity will flow, and this is represented by 1. Figure 4 shows the diagram of a two-switch series circuit , where electricity will flow from the source to the destination only if both switches are closed. This diagram represents the and condition of Boolean algebra. A circuit where electricity flows whenever at least one of the switches is closed is known as a parallel circuit. This corresponds to the or condition of Boolean algebra. Figure 5 shows the diagram of a two-switch parallel circuit. To represent the not condition, one must remember that in this system a switch has only two possible positions, open or closed. Its complement is a switch that will have the opposite position. For example, if switch A is open, its complement will be closed and vice versa. Logic designers can use these diagrams to plan complex computer circuits that will perform the needed functions for a specific machine. Information Theory Boolean algebra is used in information theory because almost all search engines allow someone to enter queries in the form of logical expressions. The operator and is used to narrow a query whereas or is used to broaden it. The operator not is used to exclude specific words from a query. For example, if one is looking for information about "privacy in computer environments," she could phrase her query as "computer and privacy," or "computer or privacy," or even "computer and privacy not mainframes. The first query will retrieve fewer documents because it will only select those that contain both terms. The second will retrieve many documents because it will select those that contain "computer," those that contain "privacy," and also those that contain both terms. The last query will retrieve documents that contain both "privacy" and "computer," while anything containing the term "mainframe" will be discarded. When using search engines, one must realize that each one will access its database differently. Typically the same search performed in more than one database will not return the same result. To do a thorough search, one must become familiar with a few of the different search engines and understand their major features, such as Boolean logic and truncation. Flynn McCullough, Robert N. Mathematics for Data Processing, 2nd ed. Boolean Algebra and Its Applications. Cite this article Pick a style below, and copy the text for your bibliography.

Chapter 6 : Boolean Algebra and Its Applications

Boolean Algebra and Its Applications. J. Eldon Whitesitt. Addison-Wesley, Reading, Mass., x + pp. Illus. $

Chapter 7 : Boolean Algebra and Its Applications by J. Eldon Whitesitt

Applications of Boolean Algebra: Claude Shannon and Circuit Design Janet Heine Barnett 22 May 1 Introduction On virtually the same day in , two major new works on logic were published by prominent.

Chapter 8 : Boolean Algebra and Its Applications - J. Eldon Whitesitt - Google Books

Boolean Algebra with its application to the development of logic circuits will provide some hands on application and to bring to students the importance of technology and the effects it has had on our society.

Chapter 9 : Boolean algebra - Wikipedia

In mathematics and mathematical logic, Boolean algebra is the branch of algebra in which the values of the variables are the truth values true and false, usually denoted 1 and 0 respectively.