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Chapter 1 : REGULAR AND CHAOTIC DYNAMICS Home Page

In fact, the pendulum is a more specific type of a Hamiltonian system. The class of Hamiltonian systems in which the pendulum fits into is called the Newtonian system.

However, a Hamiltonian formulation gives much more than just this simplification. Moreover, waves in inviscid fluids such as surface water waves or magnetohydrodynamic waves also have a Hamiltonian PDE formulation. Quantum mechanics is formally obtained from classical mechanics by replacing the canonical momentum in the Hamiltonian by a differential operator. Hamiltonian structure provides strong constraints on the flow. Similarly if the Hamiltonian is independent of one of the configuration variables the variable is ignorable , then 2 implies that the corresponding canonical momentum is an invariant. One of the stronger constraints imposed by Hamiltonian structure relates to stability: Even more structure applies: Kolmogorov, Arnold and Moser proved that a sufficiently smooth, nearly-integrable Hamiltonian system still has many such invariant tori see KAM theory. Such systems are called natural Hamiltonian systems. For example suppose that there are two masses connected to three springs as shown in Figure 1. However, the trajectories of the pendulum are easy to visualize since the energy is conserved, see Figure 3. Since the energy is conserved, the orbit must be periodic. The dynamics of three or more bodies can be extremely complex. Geometric Structure Much of the elegance of the Hamiltonian formulation stems from its geometric structure. In this case the Hamiltonian phase space is the cotangent bundle of the configuration space. Conservation of Energy If a Hamiltonian does not depend explicitly on time, then its value, the energy, is constant. In the autonomous case, a Hamiltonian system conserves energy, however, it is easy to construct nonHamiltonian systems that also conserve an energy-like quantity. This immediately implies that the volume of any bundle of trajectories is preserved. The set of linear mappings that obey 10 is called the symplectic group ; it is a Lie group. Integrable Systems A dynamical system is integrable when it can be solved in some way. One rather restrictive way in which this can happen is if the flow of the vector field can be constructed analytically. However, since this can almost never be done in terms of elementary functions , this is not an especially useful class of systems. However, there is a class of Hamiltonian systems, action-angle systems, whose solutions can be obtained analytically, and there is a well-accepted definition of integrability for Hamiltonian dynamics due to Liouville in which each integrable Hamiltonian is locally equivalent to these action-angle systems. Hamiltonian systems with two or more degrees of freedom cannot always be reduced to action-angle form, giving rise to chaotic motion. Liouville Integrability Liouville and Arnold showed that the motion in a larger class of Hamiltonian systems is as simple as that of For example, every one degree-of-freedom, autonomous Hamiltonian system is Liouville integrable. However, the action-angle coordinates may not be globally defined. In the case of the pendulum 4 , there are action-variables away from the separatrix. Generically the dynamics on an invariant torus are quasiperiodic. The method lead to theorems by Vladimir Arnold for analytic Hamiltonian systems Arnold, and by Jurgen Moser for smooth enough area-preserving mappings Moser , and the ideas have become known as KAM theory. For more details see Kolmogorov-Arnold-Moser Theory. Hamiltonian Chaos Figure 4: Though many invariant tori of an integrable system typically persist upon a perturbation, tori that commensurate or nearly commensurate are typically destroyed. Chaotic dynamics often occurs in the neighborhood of these destroyed tori. For example, consider the 1. Moreover, the stable and unstable manifolds of the saddle typically intersect transversely, giving rise to a Smale horseshoe and chaotic motion albeit chaos that is limited to a narrow layer about the separatrix. Mathematical Methods of Classical Mechanics. Introduction to Symplectic Topology. Introduction to the Theory of Hamiltonian Systems. Lectures on Celestial Mechanics. Internal references Paul M. Vitanyi Andrey Nikolaevich Kolmogorov. Scholarpedia , 2 2: Kuznetsov Conjugate maps. James Meiss Dynamical systems. Ferdinand Verhulst Hamiltonian normal forms. Shea-Brown Periodic orbit. Samoilenko Quasiperiodic oscillations. Steve Smale and Michael Shub Smale horseshoe. Philip Holmes and Eric T. Alain Chenciner Three body problem.

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Chapter 2 : Hamiltonian systems - Scholarpedia

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The output of op amp 0 will correspond to the x variable, the output of 1 corresponds to the first derivative of x and the output of 2 corresponds to the second derivative. Spontaneous order[edit] Under the right conditions, chaos spontaneously evolves into a lockstep pattern. In the Kuramoto model , four conditions suffice to produce synchronization in a chaotic system. Natural forms ferns, clouds, mountains, etc. In the s, while studying the three-body problem , he found that there can be orbits that are nonperiodic, and yet not forever increasing nor approaching a fixed point. Chaos theory began in the field of ergodic theory. Despite initial insights in the first half of the twentieth century, chaos theory became formalized as such only after mid-century, when it first became evident to some scientists that linear theory , the prevailing system theory at that time, simply could not explain the observed behavior of certain experiments like that of the logistic map. What had been attributed to measure imprecision and simple " noise " was considered by chaos theorists as a full component of the studied systems. The main catalyst for the development of chaos theory was the electronic computer. Much of the mathematics of chaos theory involves the repeated iteration of simple mathematical formulas, which would be impractical to do by hand. Electronic computers made these repeated calculations practical, while figures and images made it possible to visualize these systems. Yet his advisor did not agree with his conclusions at the time, and did not allow him to report his findings until Studies of the critical point beyond which a system creates turbulence were important for chaos theory, analyzed for example by the Soviet physicist Lev Landau , who developed the Landau-Hopf theory of turbulence. David Ruelle and Floris Takens later predicted, against Landau, that fluid turbulence could develop through a strange attractor , a main concept of chaos theory. Edward Lorenz was an early pioneer of the theory. His interest in chaos came about accidentally through his work on weather prediction in He wanted to see a sequence of data again, and to save time he started the simulation in the middle of its course. He did this by entering a printout of the data that corresponded to conditions in the middle of the original simulation. To his surprise, the weather the machine began to predict was completely different from the previous calculation. Lorenz tracked this down to the computer printout. The computer worked with 6-digit precision, but the printout rounded variables off to a 3-digit number, so a value like 0. This difference is tiny, and the consensus at the time would have been that it should have no practical effect. However, Lorenz discovered that small changes in initial conditions produced large changes in long-term outcome. In , Benoit Mandelbrot found recurring patterns at every scale in data on cotton prices. In , he published " How long is the coast of Britain? In , Mandelbrot published The Fractal Geometry of Nature , which became a classic of chaos theory. Yorke coiner of the term "chaos" as used in mathematics , Robert Shaw , and the meteorologist Edward Lorenz. In , Albert J. Feigenbaum for their inspiring achievements. There, Bernardo Huberman presented a mathematical model of the eye tracking disorder among schizophrenics. In , Per Bak , Chao Tang and Kurt Wiesenfeld published a paper in Physical Review Letters [59] describing for the first time self-organized criticality SOC , considered one of the mechanisms by which complexity arises in nature. Alongside largely lab-based approaches such as the Bakâ€”Tangâ€”Wiesenfeld sandpile , many other investigations have focused on large-scale natural or social systems that are known or suspected to display scale-invariant behavior. Although these approaches were not always welcomed at least initially by specialists in the subjects examined, SOC has nevertheless become established as a strong candidate for explaining a number of natural phenomena, including earthquakes , which, long before SOC was discovered, were known as a source of scale-invariant behavior such as the Gutenbergâ€”Richter law describing the statistical distribution of earthquake sizes, and the Omori law [60] describing the frequency of aftershocks , solar flares , fluctuations in economic systems such as financial markets references to SOC are common in econophysics , landscape formation , forest fires , landslides , epidemics , and biological evolution where SOC has been invoked, for example, as the dynamical mechanism behind the theory of " punctuated equilibria " put forward by Niles Eldredge and Stephen Jay Gould. Given the implications of a scale-free distribution of event sizes, some researchers have suggested that another phenomenon that should be considered an example of SOC is the occurrence of wars. In the same year, James Gleick published Chaos: Making a New Science , which became a best-seller and introduced the general principles of chaos theory as well as its history to the broad public, though his history under-emphasized important Soviet contributions.

Chapter 3 : Chaos theory - Wikipedia

CHAOTIC DYNAMICS is the phase space in Hamiltonian systems can be structures in a PoincarÃ© surface of section for a chaotic Hamiltonian system, and the.

Random variables are expanded in terms of orthogonal polynomials and differential equations are derived for the expansion coefficients. Here we study the structure and dynamics of these differential equations when the original system has Hamiltonian structure, multiple time scales, or chaotic dynamics. In particular, we prove that the differential equations for the coefficients in generalized polynomial chaos expansions of Hamiltonian systems retain the Hamiltonian structure relative to the ensemble average Hamiltonian. We connect this with the volume-preserving property of Hamiltonian flows to show that, for an oscillator with uncertain frequency, a finite expansion must fail at long times, regardless of truncation order. Finally, using the forced Duffing oscillator as an example, we demonstrate that when the original dynamical system displays chaotic dynamics, the resulting dynamical system from polynomial chaos also displays chaotic dynamics, limiting its applicability. Polynomial chaos based uncertainty quantification in Hamiltonian, multi-time scale, and chaotic systems. Journal of Computational Dynamics, , 1 2: Sudret, An adaptive algorithm to build up sparse polynomial chaos expansions for stochastic finite element analysis,, Probabilistic Engineering Mechanics, 25 , Sudret, Adaptive sparse polynomial chaos expansion based on least angle regression,, Journal of Computational Physics, , Martin, The orthogonal development of non-linear functionals in series of Fourier-Hermite functionals,, Annals of Mathematics, 48 , Sachse, Sensitivity analysis of cardiac electrophysiological models using polynomial chaos,, in Engineering in Medicine and Biology Society, , Ghanem, Probabilistic characterization of transport in heterogeneous media,, Comput. Wanner, Geometric Numerical Integration: Friedman, The Elements of Statistical Learning,, 2nd edition, Crutchfield, Chaotic states of anharmonic systems in periodic fields,, Phys. Dellnitz, An efficient algorithm for the parallel solution of high-dimensional differential equations,, J. Zabaras, The effect of multiple sources of uncertainty on the convex hull of material properties of polycrystals,, Computational Materials Science, 47 , Laskar, Large-scale chaos in the solar system,, Astronomy and Astrophysics, Classical, Quantum, and Relativistic Descriptions,, 2nd edition, Zabaras, An adaptive hierarchical sparse grid collocation algorithm for the solution of stochastic differential equations,, Journal of Computational Physics, , Zabaras, Kernel principal component analysis for stochastic input model generation,, Journal of Computational Physics, , Najm, Dimensionality reduction and polynomial chaos acceleration of Bayesian inference in inverse problems,, Journal of Computational Physics, , Anderson-Cook, Response Surface Methodology,, 3rd edition, Webster, A sparse grid stochastic collocation method for partial differential equations with randon input data,, SIAM J. Webster, A stochastic multiscale model for electricity generation capacity expansion,, Eur. Loire, Uncertainty as a stabilizer of the head-tail ordered phase in carbon-monoxide monolayers on graphite,, Physical Review B, 80 Sahai, Backbone transitions and invariant tori in forced micromechanical oscillators with optical detection,, Nonlinear Dynamics, 62 , Zehnder, Thermomechanical transitions in doubly-clamped micro-oscillators,, International Journal of Non-Linear Mechanics, 42 , Pasini, Uncertainty quantification in hybrid dynamical systems,, J. Banaszuk, Hearing the clusters in a graph: A dristributed algorithm,, Automatica, 48 , Zehnder, Modeling of coupled dome-shaped microoscillators,, Microelectromechanical Systems, 17 , Strogatz, Nonlinear Dynamics and Chaos: Banaszuk, Iterative methods for scalable uncertainty quantification in complex networks,, International Journal for Uncertainty Quantification, 2 , Hikihara, Coherent swing instability of power grids,, J. Hellerman , , Karniadakis, Beyond Wiener-Askey expansions: Karniadakis, Recent advances in polynomial chaos methods and extensions,, in Computational Uncertainty in Military Vehicle Design Meeting Proceedings, Karniadakis, Long-term behavior of polynomial chaos in stochastic flow simulations,, Computer methods in applied mechanics and engineering, , Wiener, The homogeneous chaos,, American Journal of Mathematics, 60 , Karniadakis, Modeling uncertainty in flow simulations via generalized polynomial chaos,, J. Ganapathysubramanian, A scalable framework for the solution of stochastic inverse problems using a sparse grid collocation approach,, J. Measuring the total amount of chaos in some Hamiltonian systems. Majda ,

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Chapter 4 : [] Kuramoto dynamics in Hamiltonian systems

The results of our tests of chaotic dynamics are not conclusive. We found some difficulties in calculating the correlation dimension due to insufficient economic data.

Chapter 5 : On Local Lyapunov Exponents of Chaotic Hamiltonian Systems | CMST

In Hamiltonian mechanics, the dynamics is deï¬•ned by the evol ution of points in phase space. For a system with n degress of freedom, the phase space coordinates are made up of n generalised.

Chapter 6 : Regular and chaotic dynamics - Allan J. Lichtenberg, Michael A. Lieberman - Google Books

The drift of the unperturbed integrals or orbital elements due to a chaotic dynamics is known as chaotic diffusion, and in general, this instability is a global property of near- integrable Hamiltonian systems with more than two degrees of freedom.