

## Chapter 1 : Applications of Combinatorial Game Theory - Mathematics Stack Exchange

*Combinatorial game theory (CGT) is a branch of mathematics and theoretical computer science that typically studies sequential games with perfect information.*

History[ edit ] CGT arose in relation to the theory of impartial games , in which any play available to one player must be available to the other as well. One such game is nim , which can be solved completely. Nim is an impartial game for two players, and subject to the normal play condition, which means that a player who cannot move loses. In the s, the Spragueâ€™Grundy theorem showed that all impartial games are equivalent to heaps in nim, thus showing that major unifications are possible in games considered at a combinatorial level, in which detailed strategies matter, not just pay-offs. In the s, Elwyn R. Berlekamp , John H. Conway and Richard K. Guy jointly introduced the theory of a partisan game , in which the requirement that a play available to one player be available to both is relaxed. Their results were published in their book *Winning Ways for your Mathematical Plays* in Combinatorial games are generally, by convention, put into a form where one player wins when the other has no moves remaining. It is easy to convert any finite game with only two possible results into an equivalent one where this convention applies. One of the most important concepts in the theory of combinatorial games is that of the sum of two games, which is a game where each player may choose to move either in one game or the other at any point in the game, and a player wins when his opponent has no move in either game. This way of combining games leads to a rich and powerful mathematical structure. John Conway states in *ONAG* that the inspiration for the theory of partisan games was based on his observation of the play in go endgames, which can often be decomposed into sums of simpler endgames isolated from each other in different parts of the board. Examples[ edit ] The introductory text *Winning Ways* introduced a large number of games, but the following were used as motivating examples for the introductory theory: Blueâ€™Red Hackenbush - At the finite level, this partisan combinatorial game allows constructions of games whose values are dyadic rational numbers. At the infinite level, it allows one to construct all real values, as well as many infinite ones that fall within the class of surreal numbers. Blueâ€™Redâ€™Green Hackenbush - Allows for additional game values that are not numbers in the traditional sense, for example, star. Toads and Frogs - Allows various game values. Unlike most other games, a position is easily represented by a short string of characters. Domineering - Various interesting games, such as hot games , appear as positions in Domineering, because there is sometimes an incentive to move, and sometimes not. Nim - An impartial game. This allows for the construction of the numbers. It can also be seen as a green-only special case of Blue-Red-Green Hackenbush. The classic game Go was influential on the early combinatorial game theory, and Berlekamp and Wolfe subsequently developed an endgame and temperature theory for it see references. Armed with this they were able to construct plausible Go endgame positions from which they could give expert Go players a choice of sides and then defeat them either way. Another game studied in the context of combinatorial game theory is chess. In Alan Turing wrote of the game, "If one can explain quite unambiguously in English, with the aid of mathematical symbols if required, how a calculation is to be done, then it is always possible to programme any digital computer to do that calculation, provided the storage capacity is adequate. Infinite chess has an even greater combinatorial complexity than chess unless only limited end-games, or composed positions with a small number of pieces are being studied. Overview[ edit ] A game, in its simplest terms, is a list of possible "moves" that two players, called left and right, can make. The game position resulting from any move can be considered to be another game. This idea of viewing games in terms of their possible moves to other games leads to a recursive mathematical definition of games that is standard in combinatorial game theory. Using Domineering as an example, label each of the sixteen boxes of the four-by-four board by A1 for the upper leftmost square, C2 for the third box from the left on the second row from the top, and so on. D3, D4 to stand for the game position in which a vertical domino has been placed in the bottom right corner.

## Chapter 2 : Combinatorial Games (Erik Demaine)

*Erik Demaine's Combinatorial Games Page Recently I have become quite interested in combinatorial game theory, particularly algorithmic combinatorial game [www.nxgvision.com](http://www.nxgvision.com) both settings, the object of interest is a combinatorial game, which usually involves complete information, with no hidden cards and no randomness--a pure strategy game.*

In both settings, the object of interest is a combinatorial game, which usually involves complete information, with no hidden cards and no randomness--a pure strategy game. In general, combinatorial game theory is a suite of techniques for analyzing such games. The algorithmic side asks for efficient algorithms for playing games optimally, or for computational intractability results suggesting that no such efficient algorithms exist. Survey Paper I recently completed a survey paper about results in algorithmic combinatorial game theory, plus a short introduction to combinatorial game theory. One main categorization of combinatorial games is how many players are involved in play. There is a large body of work on two-player games. In particular, the book *Winning Ways* by Berlekamp, Conway, and Guy, builds a beautiful theory for classifying games. Another type of combinatorial game is a one-player game, also called a puzzle. Many games in real life are essentially one-player. One-player games also arise naturally when examining a portion of a two-player game. The final main type of combinatorial game is a zero-player game. The main example of such a game is a cellular automaton such as John H. My Research I am particularly excited by one-player combinatorial games, and would like to advocate their study. Here are some combinatorial puzzles we have analyzed more information to come soon: A variety of geometric games involving the construction, transformation, or marking of the edges of a planar triangulation. We give polynomial-time winning strategies in several cases. The classic computer game in which tetriminoes pieces made up of 4 unit squares fall one at a time into a rectangular board, the player can slide the piece left or right during the fall, and any completely filled rows are erased. We prove that the offline perfect-information version with a generalized board is NP-complete under many variations on the rules and goal of the game. We analyze the solitaire version of Clobber where one player controls both sides, and the goal is to remove as many stones as possible. Sliding blocks and NCL: Sliding-block puzzles consist of a collection of rectangular blocks in a rectangular box, and the goal is to move one piece to a particular location. We prove that these puzzles are PSPACE-complete even for 1-by-2 blocks, and when the goal is just to move one piece at all. Puzzles involving a robot walking around in a square grid, pushing square-block obstacles subject to various rules, in order to reach a specified goal position. A puzzle game in which the player clicks on a connected group of two or more blocks of a common color, and blocks above fall down to take their place. The goal is to remove as many blocks as possible. A two-player game by Conway involving one black stone the ball and several white stones placed on a grid. In each turn, a player can drop a white stone on an empty grid point, or "kick" the black stone multiple times over sequences of white stones horizontally, vertically, or diagonally and remove those white stones. We analyze the complexity of deciding "mate in 1", which is a combinatorial puzzle. A wide variety of coin-sliding and coin-moving puzzles can be solved in polynomial time, leading to several new puzzles that are difficult for humans to solve but are guaranteed to be solvable by algorithmic methods. Black box You may also be interested in some puzzles we have designed for fun. References See also the survey paper mentioned at the top of this page. There are several other webpages on the topic of combinatorial games: Jeff Erickson has another excellent page on mathematical games, toys, and puzzles. The page also includes Richard K. Conway, and Richard K. The two-volume book was published by Academic Press London in Unfortunately, it is currently out-of-print, but work is underway to publish a second edition. Another excellent reference on combinatorial game theory, with a more formal mathematical slant, is *On Numbers and Games* by John H. Conway, also published by Academic Press London , Last updated November 28, by Erik Demaine.

## Chapter 3 : Combinatorial game theory - Wikipedia

*This course will cover the mathematical theory and analysis of simple games without chance moves. Hello and welcome to Games Without Chance: Combinatorial Game Theory! The topic for this first week is Let's play a game: Students will learn what a combinatorial game is, and play simple games.*

The English used in this article or section may not be easy for everybody to understand. You can help Wikipedia by reading Wikipedia: How to write Simple English pages , then simplifying the article. December Combinatorial game theory, also known as CGT is a branch of applied mathematics and theoretical computer science that studies combinatorial games, and is distinct from "traditional" or "economic" game theory. CGT arose in relation to the theory of impartial games, the two-player game of Nim in particular, with an emphasis on "solving" certain types of combinatorial games. A game must meet several conditions to be a combinatorial game. The game must have at least two players. The game must be sequential i. The game must have perfect information i. The game must be deterministic i. Luck is not a part of the game. The game must have a defined amount of possible moves. The game must eventually end. The game must end when one player can no longer move. Combinatorial Game Theory is largely confined to the study of a subset of combinatorial games which are two player, finite, and have a winner and loser i. These combinatorial games can be represented by trees, each vertex of which is the game resulting from a particular move from the game directly below it on the tree. These games can be assigned game values. Finding these game values is of great interests to CG theorists, as is the theoretical concept of game addition. They worked together in the s. Definitions[ change change source ] In the theory, there are two players called left and right. A game is something that allows left and right to make moves to other games. For example, in the game of chess , there is a usual starting setup. One could also, however, think of a chess game after the first move as a different game, with a different setup. So each position is also called a game. L are the games the left player can move to.

## Chapter 4 : Combinatorial game theory - Simple English Wikipedia, the free encyclopedia

*A combinatorial game is a two player game that satisfies the following conditions: The game is deterministic: there is no randomization mechanism such as flipping a coin or rolling a die.*

## Chapter 5 : Combinatorial | Family | BoardGameGeek

*In the normal game, the last player to move wins. In the misère variant, the last player to move is declared the loser. The theory of combinatorial games arose from that of impartial games in which, at every stage, the same set of moves is available to both players, regardless of whose move it is.*

## Chapter 6 : Combinatorial Games: Tic-Tac-Toe Theory - Jozsef Beck, József Beck - Google Books

*Combinatorial Games. Example Nim In a one pile game, we start with a  $n \neq 0$  chips and while there is a positive number  $x$  of chips, a move consists of deleting  $y \leq x$ .*

## Chapter 7 : What Is a Combinatorial Game?

*Combinatorial Games Turn-based competitive multi-player games Can be a simple win-or-lose game, or can involve points Everyone has perfect information Each turn, the player changes the current "state" using a.*

## Chapter 8 : Combinatorics - Wikipedia

*Combinatorial Game Theory studies strategies and mathematics of two-player games of perfect knowledge such as*

*chess or go (but often either concentrating instead on simpler games such as nim, or solving endgames and other special cases). An important distinction between this subject and classical.*

### Chapter 9 : Combinatorial Game Suite

*Combinatorial Game Theory Alan Chang Going back to our subtraction game, we see that the P-positions consist of all the positions which are divisible by 3, and the rest are N-positions.*