DOWNLOAD PDF EXPRESSION AND THE ORIGIN OF GEOMETRY

Chapter 1 : Glossary of Mathematical Terms - The Story of Mathematics

The History of Geometry. Geometry's origins go back to approximately 3, BC in ancient Egypt. Ancient Egyptians used an early stage of geometry in several ways, including the surveying of land, construction of pyramids, and astronomy.

Differential geometry The German mathematician Carl Friedrich Gauss â€" , in connection with practical problems of surveying and geodesy, initiated the field of differential geometry. Using differential calculus , he characterized the intrinsic properties of curves and surfaces. For instance, he showed that the intrinsic curvature of a cylinder is the same as that of a plane, as can be seen by cutting a cylinder along its axis and flattening, but not the same as that of a sphere , which cannot be flattened without distortion. Instead, they discovered that consistent non-Euclidean geometries exist. Topology Topology, the youngest and most sophisticated branch of geometry, focuses on the properties of geometric objects that remain unchanged upon continuous deformationâ€"shrinking, stretching, and folding, but not tearing. The continuous development of topology dates from , when the Dutch mathematician L. Brouwer â€" introduced methods generally applicable to the topic. History of geometry The earliest known unambiguous examples of written recordsâ€"dating from Egypt and Mesopotamia about bceâ€"demonstrate that ancient peoples had already begun to devise mathematical rules and techniques useful for surveying land areas, constructing buildings, and measuring storage containers. It concludes with a brief discussion of extensions to non-Euclidean and multidimensional geometries in the modern age. Similarly, eagerness to know the volumes of solid figures derived from the need to evaluate tribute, store oil and grain, and build dams and pyramids. Even the three abstruse geometrical problems of ancient timesâ€"to double a cube, trisect an angle, and square a circle, all of which will be discussed laterâ€"probably arose from practical matters, from religious ritual, timekeeping, and construction, respectively, in pre-Greek societies of the Mediterranean. And the main subject of later Greek geometry, the theory of conic sections , owed its general importance, and perhaps also its origin, to its application to optics and astronomy. While many ancient individuals, known and unknown, contributed to the subject, none equaled the impact of Euclid and his Elements of geometry, a book now 2, years old and the object of as much painful and painstaking study as the Bible. Much less is known about Euclid , however, than about Moses. Euclid wrote not only on geometry but also on astronomy and optics and perhaps also on mechanics and music. Only the Elements, which was extensively copied and translated, has survived intact. What is known about Greek geometry before him comes primarily from bits quoted by Plato and Aristotle and by later mathematicians and commentators. Among other precious items they preserved are some results and the general approach of Pythagoras c. The Pythagoreans convinced themselves that all things are, or owe their relationships to, numbers. The doctrine gave mathematics supreme importance in the investigation and understanding of the world. Plato developed a similar view, and philosophers influenced by Pythagoras or Plato often wrote ecstatically about geometry as the key to the interpretation of the universe. Thus ancient geometry gained an association with the sublime to complement its earthy origins and its reputation as the exemplar of precise reasoning. Finding the right angle Ancient builders and surveyors needed to be able to construct right angles in the field on demand. One way that they could have employed a rope to construct right triangles was to mark a looped rope with knots so that, when held at the knots and pulled tight, the rope must form a right triangle. The simplest way to perform the trick is to take a rope that is 12 units long, make a knot 3 units from one end and another 5 units from the other end, and then knot the ends together to form a loop, as shown in the animation. However, the Egyptian scribes have not left us instructions about these procedures, much less any hint that they knew how to generalize them to obtain the Pythagorean theorem: The required right angles were made by ropes marked to give the triads 3, 4, 5 and 5, 12, In Babylonian clay tablets c. A right triangle made at random, however, is very unlikely to have all its sides measurable by the same unitâ€"that is, every side a whole-number multiple of some common unit of measurement. This fact, which came as a shock when discovered by the Pythagoreans, gave rise to the concept and theory of incommensurability. Locating the inaccessible By ancient tradition, Thales of Miletus , who lived before Pythagoras in the 6th century bce, invented a way to measure inaccessible heights, such as the Egyptian pyramids. Although none of his writings survives, Thales may well have known about a Babylonian observation that for similar triangles triangles having the same shape but not necessarily the same size the length of each corresponding side is increased or decreased by the same multiple. A determination of the height of a tower using similar triangles is demonstrated in the figure. A comparison of a Chinese and a Greek geometric theoremThe figure illustrates the equivalence of the Chinese complementary rectangles theorem and the Greek similar triangles theorem. Estimating the wealth A Babylonian cuneiform tablet written some 3, years ago treats problems about dams, wells, water clocks, and excavations. Ahmes , the scribe who copied and annotated the Rhind papyrus c. Euclid arbitrarily restricted the tools of construction to a straightedge an unmarked ruler and a compass. The restriction made three problems of particular interest to double a cube, to trisect an arbitrary angle, and to square a circle very difficultâ€”in fact, impossible. Various methods of construction using other means were devised in the classical period, and efforts, always unsuccessful, using straightedge and compass persisted for the next 2, years. Doubling the cube The Vedic scriptures made the cube the most advisable form of altar for anyone who wanted to supplicate in the same place twice. The rules of ritual required that the altar for the second plea have the same shape but twice the volume of the first. The problem came to the Greeks together with its ceremonial content. An oracle disclosed that the citizens of Delos could free themselves of a plague merely by replacing an existing altar by one twice its size. The Delians applied to Plato. Hippocrates of Chios , who wrote an early Elements about bce, took the first steps in cracking the altar problem. He reduced the duplication to finding two mean proportionals between 1 and 2, that is, to finding lines x and y in the ratio 1: A few generations later, Eratosthenes of Cyrene c. Trisecting the angle The Egyptians told time at night by the rising of 12 asterisms constellations , each requiring on average two hours to rise. In order to obtain more convenient intervals, the Egyptians subdivided each of their asterisms into three parts, or decans. That presented the problem of trisection. It is not known whether the second celebrated problem of archaic Greek geometry, the trisection of any given angle, arose from the difficulty of the decan, but it is likely that it came from some problem in angular measure. Although no one succeeded in finding a solution with straightedge and compass, they did succeed with a mechanical device and by a trick. The mechanical device, perhaps never built, creates what the ancient geometers called a quadratrix. Invented by a geometer known as Hippias of Elis flourished 5th century bce , the quadratrix is a curve traced by the point of intersection between two moving lines, one rotating uniformly through a right angle, the other gliding uniformly parallel to itself. The Quadratrix of Hippias. The trick for trisection is an application of what the Greeks called neusis, a maneuvering of a measured length into a special position to complete a geometrical figure. A late version of its use, ascribed to Archimedes c. Squaring the circle The pre-Euclidean Greek geometers transformed the practical problem of determining the area of a circle into a tool of discovery. Three approaches can be distinguished: While not able to square the circle, Hippocrates did demonstrate the quadratures of lunes; that is, he showed that the area between two intersecting circular arcs could be expressed exactly as a rectilinear area and so raised the expectation that the circle itself could be treated similarly. Quadrature of the Lune. These were the substitution and mechanical approaches. The method of exhaustion as developed by Eudoxus approximates a curve or surface by using polygons with calculable perimeters and areas. Idealization and proof The last great Platonist and Euclidean commentator of antiquity, Proclus c. Proclus referred especially to the theorem, known in the Middle Ages as the Bridge of Asses, that in an isosceles triangle the angles opposite the equal sides are equal. The theorem may have earned its nickname from the Euclidean figure or from the commonsense notion that only an ass would require proof of so obvious a statement. The Bridge of Asses. The ancient Greek geometers soon followed Thales over the Bridge of Asses. In the 5th century bce the philosopher-mathematician Democritus c. By the time of Plato, geometers customarily proved their propositions. Their compulsion and the multiplication of theorems it produced fit perfectly with the endless questioning of Socrates and the uncompromising logic of Aristotle. Perhaps the origin, and certainly the exercise, of the peculiarly Greek method of mathematical proof should be sought in the same social setting that gave rise to the practice of philosophyâ€”that is, the Greek polis. There citizens learned the skills of a governing class, and the wealthier among them enjoyed the leisure to engage their minds as they pleased, however useless the result, while slaves attended to the necessities of life. Greek society could support the transformation of geometry from a practical art to a deductive science. Despite its rigour, however, Greek geometry does not satisfy the demands of the modern systematist. Euclid himself sometimes appeals to inferences drawn from an intuitive grasp of concepts such as point and line or inside and outside, uses superposition, and so on. It took more than 2, years to purge the Elements of what pure deductivists deemed imperfections. Of this preliminary matter, the fifth and last postulate, which states a sufficient condition that two straight lines meet if sufficiently extended, has received by far the greatest attention. In effect it defines parallelism. Many later geometers tried to prove the fifth postulate using other parts of the Elements. The first six books contain most of what Euclid delivers about plane geometry. Book VI applies the theory of proportion from Book V to similar figures and presents the geometrical solution to quadratic equations. As usual, some of it is older than Euclid. Books VIIâ€”X, which concern various sorts of numbers, especially primes, and various sorts of ratios, are seldom studied now, despite the importance of the masterful Book X, with its elaborate classification of incommensurable magnitudes, to the later development of Greek geometry. XI contains theorems about the intersection of planes and of lines and planes and theorems about the volumes of parallelepipeds solids with parallel parallelograms as opposite faces ; XII applies the method of exhaustion introduced by Eudoxus to the volumes of solid figures, including the sphere; XIII, a three-dimensional analogue to Book IV, describes the Platonic solids. Among the jewels in Book XII is a proof of the recipe used by the Egyptians for the volume of a pyramid. Gnomonics and the cone During its daily course above the horizon the Sun appears to describe a circular arc. Our astronomer, using the pointer of a sundial, known as a gnomon, as his eye, would generate a second, shadow cone spreading downward. The possible intersections of a plane with a cone, known as the conic sections , are the circle, ellipse, point, straight line, parabola , and hyperbola. Doubtless, however, both knew that all the conics can be obtained from the same right cone by allowing the section at any angle. Apollonius reproduced known results much more generally and discovered many new properties of the figures. He first proved that all conics are sections of any circular cone, right or oblique. Apollonius introduced the terms ellipse, hyperbola, and parabola for curves produced by intersecting a circular cone with a plane at an angle less than, greater than, and equal to, respectively, the opening angle of the cone. Astronomy and trigonometry Calculation In an inspired use of their geometry, the Greeks did what no earlier people seems to have done: Thus they assigned to the Sun a circle eccentric to the Earth to account for the unequal lengths of the seasons. Ptolemy flourished â€” ce in Alexandria, Egypt worked out complete sets of circles for all the planets. Contrary to the Elements, however, the Almagest deploys geometry for the purpose of calculation. Among the items Ptolemy calculated was a table of chords, which correspond to the trigonometric sine function later introduced by Indian and Islamic mathematicians. The table of chords assisted the calculation of distances from angular measurements as a modern astronomer might do with the law of sines.

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In mathematics, an expression or mathematical expression is a finite combination of symbols that is well-formed according to rules that depend on the context. Mathematical symbols can designate numbers (constants), variables, operations, functions, brackets, punctuation, and grouping to help determine order of operations, and other.

For example, an angle was defined as the inclination of two straight lines, and a circle was a plane figure consisting of all points that have a fixed distance radius from a given centre. Stated in modern terms, the axioms are as follows: Given two points, there is a straight line that joins them. A straight line segment can be prolonged indefinitely. A circle can be constructed when a point for its centre and a distance for its radius are given. All right angles are equal. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, will meet on that side on which the angles are less than the two right angles. Hilbert refined axioms 1 and 5 as follows: For any two different points, a there exists a line containing these two points, and b this line is unique. For any line L and point p not on L, a there exists a line through p not meeting L, and b this line is unique. It also attracted great interest because it seemed less intuitive or self-evident than the others. All five axioms provided the basis for numerous provable statements, or theorems, on which Euclid built his geometry. The rest of this article briefly explains the most important theorems of Euclidean plane and solid geometry. Plane geometry Congruence of triangles Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur. The first such theorem is the side-angle-side SAS theorem: If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, the triangles are congruent. The first very useful theorem derived from the axioms is the basic symmetry property of isosceles trianglesâ€"i. For an illustrated exposition of the proof, see Sidebar: The Bridge of Asses. The Bridge of Asses opens the way to various theorems on the congruence of triangles. The parallel postulate is fundamental for the proof of the theorem that the sum of the angles of a triangle is always degrees. A simple proof of this theorem was attributed to the Pythagoreans. Proof that the sum of the angles in a triangle is degrees. Similarity of triangles As indicated above, congruent figures have the same shape and size. Similar figures, on the other hand, have the same shape but may differ in size. Shape is intimately related to the notion of proportion, as ancient Egyptian artisans observed long ago. Segments of lengths a, b, c, and d are said to be proportional if a: The formula in the figure reads k is to l as m is to n if and only if line DE is parallel to line AB. This theorem then enables one to show that the small and large triangles are similar. The similarity theorem may be reformulated as the AAA angle-angle-angle similarity theorem: Two similar triangles are related by a scaling or similarity factor s: In addition to the ubiquitous use of scaling factors on construction plans and geographic maps, similarity is fundamental to trigonometry. Areas Just as a segment can be measured by comparing it with a unit segment, the area of a polygon or other plane figure can be measured by comparing it with a unit square. The common formulas for calculating areas reduce this kind of measurement to the measurement of certain suitable lengths. The simplest case is a rectangle with sides a and b, which has area ab. One can then compute the area of a general polygon by dissecting it into triangular regions. If a triangle or more general figure has area A, a similar triangle or figure with a scaling factor of s will have an area of s2A. For an arbitrary triangle, the Pythagorean theorem is generalized to the law of cosines: Since Euclid, a host of professional and amateur mathematicians even U. President James Garfield have found more than distinct proofs of the Pythagorean theorem. Despite its antiquity, it remains one of the most important theorems in mathematics. It enables one to calculate distances or, more important, to define distances in situations far more general than elementary geometry. For example, it has been generalized to multidimensional vector spaces. Circles A chord AB is a segment in the interior of a circle connecting two points A and B on the circumference. The Greek mathematician Archimedes c. A semicircle has its end points on a diameter of a circle. Another important theorem states that for any chord AB in a circle, the angle subtended by any point on the same semiarc of the circle will be invariant. Slightly modified, this means that in a circle, equal chords determine equal angles, and vice versa. Thales of Miletus fl.

Summarizing the above material, the five most important theorems of plane Euclidean geometry are: Most of the more advanced theorems of plane Euclidean geometry are proved with the help of these theorems. Regular polygons A polygon is called regular if it has equal sides and angles. Thus, a regular triangle is an equilateral triangle, and a regular quadrilateral is a square. A general problem since antiquity has been the problem of constructing a regular n-gon, for different n, with only ruler and compass. For example, Euclid constructed a regular pentagon by applying the above-mentioned five important theorems in an ingenious combination. Techniques, such as bisecting the angles of known constructions, exist for constructing regular n-gons for many values, but none is known for the general case. In , following centuries without any progress, Gauss surprised the mathematical community by discovering a construction for the gon. Because it is not known in general which F k are prime, the construction problem for regular n-gons is still open. Three other unsolved construction problems from antiquity were finally settled in the 19th century by applying tools not available to the Greeks. Comparatively simple algebraic methods showed that it is not possible to trisect an angle with ruler and compass or to construct a cube with a volume double that of a given cube. Showing that it is not possible to square a circle i. The three classical problems. Conic sections and geometric art The most advanced part of plane Euclidean geometry is the theory of the conic sections the ellipse , the parabola , and the hyperbola. Much as the Elements displaced all other introductions to geometry, the Conics of Apollonius of Perga c. Medieval Islamic artists explored ways of using geometric figures for decoration. In the 20th century, internationally renowned artists such as Josef Albers , Max Bill , and Sol LeWitt were inspired by motifs from Euclidean geometry. Some concepts, such as proportions and angles, remain unchanged from plane to solid geometry. For other familiar concepts, there exist analogiesâ€"most noticeably, volume for area and three-dimensional shapes for two-dimensional shapes sphere for circle, tetrahedron for triangle, box for rectangle. However, the theory of tetrahedra is not nearly as rich as it is for triangles. Active research in higher-dimensional Euclidean geometry includes convexity and sphere packings and their applications in cryptology and crystallography see crystal: Volume As explained above, in plane geometry the area of any polygon can be calculated by dissecting it into triangles. A similar procedure is not possible for solids. In the German mathematician Max Dehn showed that there exist a cube and a tetrahedron of equal volume that cannot be dissected and rearranged into each other. This means that calculus must be used to calculate volumes for even many simple solids such as pyramids. Regular solids Regular polyhedra are the solid analogies to regular polygons in the plane. Regular polygons are defined as having equal congruent sides and angles. In analogy , a solid is called regular if its faces are congruent regular polygons and its polyhedral angles angles at which the faces meet are congruent. This concept has been generalized to higher-dimensional coordinate Euclidean spaces. Whereas in the plane there exist in theory infinitely many regular polygons, in three-dimensional space there exist exactly five regular polyhedra. These are known as the Platonic solids: The five Platonic solidsThese are the only geometric solids whose faces are composed of regular, identical polygons. Placing the cursor on each figure will show it in animation. In four-dimensional space there exist exactly six regular polytopes , five of them generalizations from three-dimensional space. In any space of more than four dimensions, there exist exactly three regular polytopesâ€"the generalizations of the tetrahedron, the cube, and the octahedron. Benno Artmann Calculating areas and volumes The table presents mathematical formulas for calculating the areas of various plane figures and the volumes of various solid figures.

Chapter 3 : Introduction to Geometry | Wyzant Resources

This is a list of symbols used in all branches of mathematics to express a formula or to represent a constant.. A mathematical concept is independent of the symbol chosen to represent it.

Chapter 4 : Expression (mathematics) - Wikipedia

GENERAL FIELD OF "Origin of Geometry": The last part of Husserl's career is devoted to an analysis of a "Crisis of

European Humanity," as the title of the "Vienna Lecture" would have it. Crisis is the phenomenon of.

Chapter 5 : Math Term Definition

The expression x^2 - 3xy+ 2y^2 has 3 terms: x^2 is the first term with a coefficient of 1; -3xy is the second term, with a coefficient of -3; 2y^2 is the last term with a coefficient of www.nxgvision.com

Chapter 6 : A Maths Dictionary for Kids Interactive by Jenny Eather

Definition Of Origin. The point of intersection of the altitudes of a triangle is called an orthocenter. More About Origin. The coordinates of the origin are (0, 0).

Chapter 7 : Geometry | Definition of Geometry by Merriam-Webster

MA Fall 1 The Origins of Geometry Introduction In the beginning geometry was a collection of rules for computing lengths, areas.

Chapter 8 : What does math mean? definition, meaning and pronunciation (Free English Language Diction

Geometry definition, the branch of mathematics that deals with the deduction of the properties, measurement, and relationships of points, lines, angles, and figures in space from their defining conditions by means of certain assumed properties of space.

Chapter 9 : Definition and examples of Origin | Define Origin - Geometry - Free Math Dictionary Online

Euclidean geometry, the study of plane and solid figures on the basis of axioms and theorems employed by the Greek mathematician Euclid (c. bce).In its rough outline, Euclidean geometry is the plane and solid geometry commonly taught in secondary schools.