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Chapter 1 : Rules of calculus - functions of one variable

A solid background in advanced calculus is a prerequisite. Topics include elementary and convergence theories of convolution quotients, differential equations involving operator functions, and exponential functions of operators.

History of calculus Modern calculus was developed in 17th-century Europe by Isaac Newton and Gottfried Wilhelm Leibniz independently of each other, first publishing around the same time but elements of it appeared in ancient Greece, then in China and the Middle East, and still later again in medieval Europe and in India. Ancient[edit] Archimedes used the method of exhaustion to calculate the area under a parabola. The ancient period introduced some of the ideas that led to integral calculus, but does not seem to have developed these ideas in a rigorous and systematic way. Calculations of volume and area , one goal of integral calculus, can be found in the Egyptian Moscow papyrus 13th dynasty , c. He used the results to carry out what would now be called an integration of this function, where the formulae for the sums of integral squares and fourth powers allowed him to calculate the volume of a paraboloid. Madhava of Sangamagrama and the Kerala School of Astronomy and Mathematics thereby stated components of calculus. A complete theory encompassing these components is now well known in the Western world as the Taylor series or infinite series approximations. I think it defines more unequivocally than anything else the inception of modern mathematics, and the system of mathematical analysis, which is its logical development, still constitutes the greatest technical advance in exact thinking. Pierre de Fermat , claiming that he borrowed from Diophantus , introduced the concept of adequality , which represented equality up to an infinitesimal error term. Isaac Newton developed the use of calculus in his laws of motion and gravitation. The product rule and chain rule , [15] the notions of higher derivatives and Taylor series , [16] and of analytic functions [citation needed] were introduced by Isaac Newton in an idiosyncratic notation which he used to solve problems of mathematical physics. In his works, Newton rephrased his ideas to suit the mathematical idiom of the time, replacing calculations with infinitesimals by equivalent geometrical arguments which were considered beyond reproach. He used the methods of calculus to solve the problem of planetary motion, the shape of the surface of a rotating fluid, the oblateness of the earth, the motion of a weight sliding on a cycloid , and many other problems discussed in his Principia Mathematica In other work, he developed series expansions for functions, including fractional and irrational powers, and it was clear that he understood the principles of the Taylor series. He did not publish all these discoveries, and at this time infinitesimal methods were still considered disreputable. Gottfried Wilhelm Leibniz was the first to state clearly the rules of calculus. These ideas were arranged into a true calculus of infinitesimals by Gottfried Wilhelm Leibniz , who was originally accused of plagiarism by Newton. His contribution was to provide a clear set of rules for working with infinitesimal quantities, allowing the computation of second and higher derivatives, and providing the product rule and chain rule , in their differential and integral forms. Unlike Newton, Leibniz paid a lot of attention to the formalism, often spending days determining appropriate symbols for concepts. Today, Leibniz and Newton are usually both given credit for independently inventing and developing calculus. Newton was the first to apply calculus to general physics and Leibniz developed much of the notation used in calculus today. The basic insights that both Newton and Leibniz provided were the laws of differentiation and integration, second and higher derivatives, and the notion of an approximating polynomial series. When Newton and Leibniz first published their results, there was great controversy over which mathematician and therefore which country deserved credit. Newton derived his results first later to be published in his Method of Fluxions , but Leibniz published his " Nova Methodus pro Maximis et Minimis " first. Newton claimed Leibniz stole ideas from his unpublished notes, which Newton had shared with a few members of the Royal Society. This controversy divided English-speaking mathematicians from continental European mathematicians for many years, to the detriment of English mathematics. It is Leibniz, however, who gave the new discipline its name. Newton called his calculus " the science of fluxions ". Since the time of Leibniz and Newton, many mathematicians have contributed to the continuing development of calculus. One of the first and most complete works on both infinitesimal and integral calculus was written in by Maria Gaetana Agnesi. In early calculus the use of infinitesimal quantities was thought unrigorous, and was fiercely criticized by a number of authors, most notably Michel Rolle and Bishop Berkeley. Berkeley famously described infinitesimals as the ghosts of departed quantities in his book The Analyst in Working out a rigorous foundation for calculus occupied mathematicians for much of the century following Newton and Leibniz, and is still to some extent an active area of research today. Several mathematicians, including Maclaurin , tried to prove the soundness of using infinitesimals, but it would not be until years later when, due to the work of Cauchy and Weierstrass , a way was finally found to avoid mere "notions" of infinitely small quantities. Following the work of Weierstrass, it eventually became common to base calculus on limits instead of infinitesimal quantities, though the subject is still occasionally called "infinitesimal calculus". Bernhard Riemann used these ideas to give a precise definition of the integral. It was also during this period that the ideas of calculus were generalized to Euclidean space and the complex plane. In modern mathematics, the foundations of calculus are included in the field of real analysis , which contains full definitions and proofs of the theorems of calculus. The reach of calculus has also been greatly extended. Henri Lebesgue invented measure theory and used it to define integrals of all but the most pathological functions. Laurent Schwartz introduced distributions , which can be used to take the derivative of any function whatsoever. Limits are not the only rigorous approach to the foundation of calculus. The resulting numbers are called hyperreal numbers , and they can be used to give a Leibniz-like development of the usual rules of calculus. There is also smooth infinitesimal analysis , which differs from non-standard analysis in that it mandates neglecting higher power infinitesimals during derivations. Significance[edit] While many of the ideas of calculus had been developed earlier in Greece , China , India , Iraq, Persia , and Japan , the use of calculus began in Europe, during the 17th century, when Isaac Newton and Gottfried Wilhelm Leibniz built on the work of earlier mathematicians to introduce its basic principles. The development of calculus was built on earlier concepts of instantaneous motion and area underneath curves. Applications of differential calculus include computations involving velocity and acceleration , the slope of a curve, and optimization. Applications of integral calculus include computations involving area, volume , arc length , center of mass , work , and pressure. More advanced applications include power series and Fourier series. Calculus is also used to gain a more precise understanding of the nature of space, time, and motion. For centuries, mathematicians and philosophers wrestled with paradoxes involving division by zero or sums of infinitely many numbers. These questions arise in the study of motion and area. The ancient Greek philosopher Zeno of Elea gave several famous examples of such paradoxes. Calculus provides tools, especially the limit and the infinite series , that resolve the paradoxes.

Chapter 2 : Generalized function - Wikipedia

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This is an open access article distributed under the Creative Commons Attribution License , which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. Abstract Motivated essentially by the success of the applications of the Mittag-Leffler functions in many areas of science and engineering, the authors present, in a unified manner, a detailed account or rather a brief survey of the Mittag-Leffler function, generalized Mittag-Leffler functions, Mittag-Leffler type functions, and their interesting and useful properties. Mittag-Leffler functions in certain areas of physical and applied sciences are also demonstrated. During the last two decades this function has come into prominence after about nine decades of its discovery by a Swedish Mathematician Mittag-Leffler, due to the vast potential of its applications in solving the problems of physical, biological, engineering, and earth sciences, and so forth. In this survey paper, nearly all types of Mittag-Leffler type functions existing in the literature are presented. An attempt is made to present nearly an exhaustive list of references concerning the Mittag-Leffler functions to make the reader familiar with the present trend of research in Mittag-Leffler type functions and their applications. Introduction and its general form with being the set of complex numbers are called Mittag-Leffler functions [1 , Section The former was introduced by Mittag-Leffler [2 , 3], in connection with his method of summation of some divergent series. In his papers [2 â€" 4], he investigated certain properties of this function. Function defined by 1. In particular, functions 1. The ordinary and generalized Mittag-Leffler functions interpolate between a purely exponential law and power-law-like behavior of phenomena governed by ordinary kinetic equations and their fractional counterparts, see Lang [11 , 12], Hilfer [13 , 14], and Saxena [15]. The Mittag-Leffler function is not given in the tables of Laplace transforms, where it naturally occurs in the derivation of the inverse Laplace transform of the functions of the type , where is the Laplace transform parameter and and are constants. This function also occurs in the solution of certain boundary value problems involving fractional integrodifferential equations of Volterra type [16]. During the various developments of fractional calculus in the last four decades this function has gained importance and popularity on account of its vast applications in the fields of science and engineering. Hille and Tamarkin [17] have presented a solution of the Abel-Volterra type equation in terms of Mittag-Leffler function. During the last 15 years the interest in Mittag-Leffler function and Mittag-Leffler type functions is considerably increased among engineers and scientists due to their vast potential of applications in several applied problems, such as fluid flow, rheology, diffusive transport akin to diffusion, electric networks, probability, and statistical distribution theory. For a detailed account of various properties, generalizations, and application of this function, the reader may refer to earlier important works of Blair [18], Torvik and Bagley [19], Caputo and Mainardi [20], Dzherbashyan [10], Gorenflo and Vessella [21], Gorenflo and Rutman [22], Kilbas and Saigo [23], Gorenflo and Luchko [24], Gorenflo and Mainardi [25 , 26], Mainardi and Gorenflo [27 , 28], Gorenflo et al. This paper is organized as follows: Section 2 deals with special cases of. Functional relations of Mittag-Leffler functions are presented in Section 3. Section 4 gives the basic properties. Section 5 is devoted to the derivation of recurrence relations for Mittag-Leffler functions. In Section 6 , asymptotic expansions of the Mittag-Leffler functions are given. Integral representations of Mittag-Leffler functions are given in Section 7. Section 8 deals with the -function and its special cases. Relations of Mittag-Leffler functions with Riemann-Liouville fractional calculus operators are derived in Section Generalized Mittag-Leffler functions and some of their properties are given in Section Laplace transform, Fourier transform, and fractional integrals and derivatives are discussed in Section Section 13 is devoted to the application of Mittag-Leffler function in fractional kinetic equations. In Section 14 , time-fractional diffusion equation is solved. Solution of space-fractional diffusion equation is discussed in

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Section In Section 16 , solution of a fractional reaction-diffusion equation is investigated in terms of the -function. Section 17 is devoted to the application of generalized Mittag-Leffler functions in nonlinear waves. Recent generalizations of Mittag-Leffler functions are discussed in Section

Chapter 3 : Mathematics Books | Math Books

In mathematics, generalized functions, or distributions, are objects extending the notion of www.nxgvision.com is more than one recognized theory. Generalized functions are especially useful in making discontinuous functions more like smooth functions, and describing discrete physical phenomena such as point charges.

This is an open access article distributed under the Creative Commons Attribution License , which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. Abstract A new operational matrix of fractional integration of arbitrary order for generalized Laguerre polynomials is derived. The fractional integration is described in the Riemann-Liouville sense. This operational matrix is applied together with generalized Laguerre tau method for solving general linear multiterm fractional differential equations FDEs. The method has the advantage of obtaining the solution in terms of the generalized Laguerre parameter. In addition, only a small dimension of generalized Laguerre operational matrix is needed to obtain a satisfactory result. Illustrative examples reveal that the proposed method is very effective and convenient for linear multiterm FDEs on a semi-infinite interval. Introduction The problems of FDEs arise in various areas of science and engineering. In particular, multiterm fractional differential equations have been used to model various types of viscoelastic damping see, e. In the last few decades both theory and numerical analysis of FDEs have received an increasing attention see, e. Spectral methods are a class of techniques used in applied mathematics and scientific computing to numerically solve some differential equations. The main idea is to write the solution of the differential equation as a sum of certain orthogonal polynomial and then obtain the coefficients in the sum in order to satisfy the differential equation. Due to high-order accuracy, spectral methods have gained increasing popularity for several decades, particularly in the field of computational fluid dynamics see, e. The usual spectral methods are only available for bounded domains for solving FDEs; see [25 â€” 28]. However, it is also interesting to consider spectral methods for FDEs on the half line. Several authors developed the generalized Laguerre spectral method for the half line for ordinary, partial, and delay differential equations; see [29 â€” 31]. Recently, Saadatmandi and Dehghan [25] have proposed an operational Legendre-tau technique for the numerical solution of multiterm FDEs. The same technique based on operational matrix of Chebyshev polynomials has been used for the same problem see [32]. In [33], Doha et al. More recently, Bhrawy and Alofi [34] proposed the operational Chebyshev matrix of fractional integration in the Riemann-Liouville sense which was applied together with spectral tau method for solving linear FDEs. The operational matrix of integer integration has been determined for several types of orthogonal polynomials, such as Chebyshev polynomials [35], Legendre polynomials [36], and Laguerre and Hermite [37]. Recently, Singh et al. Till now, and to the best of our knowledge, most of formulae corresponding to those mentioned previously are unknown and are traceless in the literature for fractional integration for generalized Laguerre polynomials in the Riemann-Liouville sense. This partially motivates our interest in operational matrix of fractional integration for generalized Laguerre polynomials. Another motivation is concerned with the direct solution techniques for solving the integrated forms of FDEs on the half line using generalized Laguerre tau method based on operational matrix of fractional integration in the Riemann-Liouville sense. Finally, the accuracy of the proposed algorithm is demonstrated by test problems. The paper is organized as follows. In the next section, we introduce some necessary definitions. In Section 3 the generalized Laguerre operational matrix of fractional integration is derived. In Section 4 we develop the generalized Laguerre operational matrix of fractional integration for solving linear multiorder FDEs. In Section 5 the proposed method is applied to two examples. Some Basic Preliminaries The most used definition of fractional integration is due to Riemann-Liouville, which is defined as The operator The next equation defines the Riemann-Liouville fractional derivative of order:

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Chapter 4 : Calculus - Wikipedia

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This includes the case where B is the Fenchel transform, or more generally a Moreau conjugacy. We characterize the existence and uniqueness of a solution f in terms of generalized subdifferentials We characterize the existence and uniqueness of a solution f in terms of generalized subdifferentials, which extends K. Show Context Citation Context We show that the existence Section 3 and uniqueness Section 4 of the solution of P are characterised in terms of coverings and minimal coverings by sets which are inverses of subdifferentials We complete this list by some new proposals. We compare these specific subdifferentials to some all-purpose subdifferentials used in nonsmooth analysis. We give some hints about their uses. We also point out links with duality theories. Ishii , " This paper extends these formulas to initial func This paper extends these formulas to initial functions g which are only lower semicontinuous lsc , and possibly infinite. It is proved that the Lax formulas give a lsc viscosity solution, and the Hopf formulas result in the minimal supersolution. A level set approach is used to give the most general results. USA , enb math. The next theorem characterizes the lsc function u as the minimal supersolution of 3. We shall need the following assumption c. Under the assumptions of Th Our results allow us to derive large deviation type results in stochastic optimal control from the convergence of generalised logarithmic moment generating functions. They rely on the characterisation of the uniqueness of They rely on the characterisation of the uniqueness of the solutions of max-plus linear equations. We give an illustration for a simple investment model, in which logarithmic moment generating functions represent risk-sensitive values. Some remarks on the class of continuous semi-strictly quasiconvex functions by Aris Daniilidis, Yboon Garcia Ramos - J. We introduce the notion of variational semi- strict qua-simonotonicity for a multivalued operator T: We use this notion to characterize the subdifferentials of continuous semi- strictly We use this notion to characterize the subdifferentials of continuous semi- strictly quasiconvex functions. The proposed defi-nition is a relaxation of the standard definition of semi- strict qua-simonotonicity, the latter being appropriate only for operators with nonempty values. Thus, the derived results are extensions to the con-tinuous case of corresponding results for locally Lipschitz functions. Comparing with the convex case, a quasiconvex function might have local minima which are not global typical failure in almost all deterministic global optimization methods , or c Unilateral analysis and duality by Jean-paul Penot " We use the ideas of generalized convexity. We focus our attention on the set of multipliers. We look for an interpretation of multipliers as generalized derivatives of the performance function associated with a dualizing parameterization of the given problem.

Chapter 5 : Do Calculusâ€”Wolfram Language Documentation

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Rules of calculus - functions of one variable Derivatives: There are many different ways to indicate the operation of differentiation, also known as finding or taking the derivative. The choice of notation depends on the type of function being evaluated and upon personal preference. Suppose you have a general function: All of the following notations can be read as "the derivative of y with respect to x" or less formally, "the derivative of the function. Suppose we have the function: After applying the rules of differentiation , we end up with the following result: How do we interpret this? For example, suppose you would like to know the slope of y when the variable x takes on a value of 2. Now for the practical part. How do we actually determine the function of the slope? Almost all functions you will see in economics can be differentiated using a fairly short list of rules or formulas, which will be presented in the next several sections. How to apply the rules of differentiation Once you understand that differentiation is the process of finding the function of the slope, the actual application of the rules is straightforward. First, some overall strategy. The rules are applied to each term within a function separately. Then the results from the differentiation of each term are added together, being careful to preserve signs. Coefficients and signs must be correctly carried through all operations, especially in differentiation. The rules of differentiation are cumulative, in the sense that the more parts a function has, the more rules that have to be applied. Take the simple function: The derivative of any constant term is 0, according to our first rule. This makes sense since slope is defined as the change in the y variable for a given change in the x variable. Suppose x goes from 10 to 11; y is still equal to 15 in this function, and does not change, therefore the slope is 0. Note that this function graphs as a horizontal line. The next rule states that when the x is to the power of one, the slope is the coefficient on that x. This continues to make sense, since a change in x is multiplied by 2 to determine the resulting change in y. We add this to the derivative of the constant, which is 0 by our previous rule, and the slope of the total function is 2. Now, suppose that the variable is carried to some higher power. The power rule combined with the coefficient rule is used as follows: Therefore, the derivative of 5x3 is equal to 5 3 x 3 - 1 ; simplify to get 15x2. Add to the derivative of the constant which is 0, and the total derivative is 15x2. These rules cover all polynomials, and now we add a few rules to deal with other types of nonlinear functions. It is not as obvious why the application of the rest of the rules still results in finding a function for the slope, and in a regular calculus class you would prove this to yourself repeatedly. The most important step for the remainder of the rules is to properly identify the form, or how the terms are combined, and then the application of the rule is straightforward. For functions that are sums or differences of terms, we can formalize the strategy above as follows: Read this rule as: If the total function is f minus g, then the derivative is the derivative of the f term minus the derivative of the g term. The most straightforward approach would be to multiply out the two terms, then take the derivative of the resulting polynomial according to the above rules. Or you have the option of applying the following rule. Read this as follows: The quotient rule is similarly applied to functions where the f and g terms are a quotient. Then follow this rule:

Chapter 6 : On a Generalized Laguerre Operational Matrix of Fractional Integration

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These were disconnected aspects of mathematical analysis at the time. The intensive use of the Laplace transform in engineering led to the heuristic use of symbolic methods, called operational calculus. Since justifications were given that used divergent series , these methods had a bad reputation from the point of view of pure mathematics. They are typical of later application of generalized function methods. When the Lebesgue integral was introduced, there was for the first time a notion of generalized function central to mathematics. That means its value at a given point is in a sense not its most important feature. In functional analysis a clear formulation is given of the essential feature of an integrable function, namely the way it defines a linear functional on other functions. This allows a definition of weak derivative. During the late s and s further steps were taken, basic to future work. The Dirac delta function was boldly defined by Paul Dirac an aspect of his scientific formalism ; this was to treat measures , thought of as densities such as charge density like genuine functions. Sergei Sobolev , working in partial differential equation theory , defined the first adequate theory of generalized functions, from the mathematical point of view, in order to work with weak solutions of partial differential equations. It can be called a principled theory, based on duality theory for topological vector spaces. This now enters the theory as mollifier theory. In other words, distributions cannot be multiplied except for very special cases: For example it is not meaningful to square the Dirac delta function. Work of Schwartz from around showed that was an intrinsic difficulty. Some solutions to the multiplication problem have been proposed. One is based on a very simple and intuitive definition a generalized function given by Yu. Another solution of the multiplication problem is dictated by the path integral formulation of quantum mechanics. This fixes all products of generalized functions as shown by H. Shirokov [7] and those by E. In the second case, the algebra is constructed as multiplication of distributions. Both cases are discussed below. Non-commutative algebra of generalized functions[edit] The algebra of generalized functions can be built-up with an appropriate procedure of projection of a function.

Chapter 7 : Operational Calculus and Generalized Functions

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Chapter 8 : Mittag-Leffler Functions and Their Applications

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Chapter 9 : TraditionalForm Reference Informationâ€”Wolfram Language Documentation

Operational Calculus, Volume II is a methodical presentation of operational calculus. An outline of the general theory of linear differential equations with constant coefficients is presented. Integral operational calculus and advanced topics in operational calculus, including locally integrable functions and convergence in the space of.