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Chapter 1 : Contents: Dynamic Programming and Optimal Control

An Introduction to Mathematical Optimal Control Theory Version By Lawrence C. Evans Department of Mathematics University of California, Berkeley.

Multi-objective optimization Adding more than one objective to an optimization problem adds complexity. For example, to optimize a structural design, one would desire a design that is both light and rigid. When two objectives conflict, a trade-off must be created. There may be one lightest design, one stiffest design, and an infinite number of designs that are some compromise of weight and rigidity. The set of trade-off designs that cannot be improved upon according to one criterion without hurting another criterion is known as the Pareto set. The curve created plotting weight against stiffness of the best designs is known as the Pareto frontier. A design is judged to be "Pareto optimal" equivalently, "Pareto efficient" or in the Pareto set if it is not dominated by any other design: If it is worse than another design in some respects and no better in any respect, then it is dominated and is not Pareto optimal. The choice among "Pareto optimal" solutions to determine the "favorite solution" is delegated to the decision maker. In other words, defining the problem as multi-objective optimization signals that some information is missing: In some cases, the missing information can be derived by interactive sessions with the decision maker. Multi-objective optimization problems have been generalized further into vector optimization problems where the partial ordering is no longer given by the Pareto ordering. Multi-modal optimization[edit] Optimization problems are often multi-modal; that is, they possess multiple good solutions. They could all be globally good same cost function value or there could be a mix of globally good and locally good solutions. Obtaining all or at least some of the multiple solutions is the goal of a multi-modal optimizer. Classical optimization techniques due to their iterative approach do not perform satisfactorily when they are used to obtain multiple solutions, since it is not guaranteed that different solutions will be obtained even with different starting points in multiple runs of the algorithm. Evolutionary algorithms , however, are a very popular approach to obtain multiple solutions in a multi-modal optimization task. Classification of critical points and extrema[edit] Feasibility problem[edit] The satisfiability problem , also called the feasibility problem, is just the problem of finding any feasible solution at all without regard to objective value. This can be regarded as the special case of mathematical optimization where the objective value is the same for every solution, and thus any solution is optimal. Many optimization algorithms need to start from a feasible point. One way to obtain such a point is to relax the feasibility conditions using a slack variable ; with enough slack, any starting point is feasible. Then, minimize that slack variable until slack is null or negative. Existence[edit] The extreme value theorem of Karl Weierstrass states that a continuous real-valued function on a compact set attains its maximum and minimum value. More generally, a lower semi-continuous function on a compact set attains its minimum; an upper semi-continuous function on a compact set attains its maximum. More generally, they may be found at critical points , where the first derivative or gradient of the objective function is zero or is undefined, or on the boundary of the choice set. Optima of equality-constrained problems can be found by the Lagrange multiplier method. Sufficient conditions for optimality[edit] While the first derivative test identifies points that might be extrema, this test does not distinguish a point that is a minimum from one that is a maximum or one that is neither. When the objective function is twice differentiable, these cases can be distinguished by checking the second derivative or the matrix of second derivatives called the Hessian matrix in unconstrained problems, or the matrix of second derivatives of the objective function and the constraints called the bordered Hessian in constrained problems. If a candidate solution satisfies the first-order conditions, then satisfaction of the second-order conditions as well is sufficient to establish at least local optimality. Sensitivity and continuity of optima[edit] The envelope theorem describes how the value of an optimal solution changes when an underlying parameter changes. The process of computing this change is called comparative statics. The maximum theorem of Claude Berge describes the continuity of an optimal solution as a function of underlying parameters. Calculus of optimization[edit] See also: More generally, a zero subgradient certifies that a local minimum has been found for minimization problems with convex functions and other locally Lipschitz functions. Further, critical points can be classified using the definiteness of the Hessian matrix: If the Hessian is positive definite at a critical point, then the point is a local minimum; if the Hessian matrix is negative definite, then the point is a local maximum; finally, if indefinite, then the point is some kind of saddle point. Constrained problems can often be transformed into unconstrained problems with the help of Lagrange multipliers. Lagrangian relaxation can also provide approximate solutions to difficult constrained problems. When the objective function is convex , then any local minimum will also be a global minimum. There exist efficient numerical techniques for minimizing convex functions, such as interior-point methods. Computational optimization techniques[edit] To solve problems, researchers may use algorithms that terminate in a finite number of steps, or iterative methods that converge to a solution on some specified class of problems , or heuristics that may provide approximate solutions to some problems although their iterates need not converge.

Chapter 2 : Dynamic Programming for Optimal Control Problems in Economics | Mathematics of Planet Ea

mathematical programming method which has been used success fully for over four years. The basic idea is a straightforward extension of static programming techniques already familiar to economists.

Over the years, as more difficult applied problems have been attacked, the theory has made advances. Here we consider the optimal control theory of infinite dimensional systems that has recently found interesting applications in theoretical economics so that economic models can be made more realistic. These infinite dimensional systems are usually dynamical systems whose evolution is described by a partial differential equation PDE or a delay differential equation DDE. Here we discuss the use of the Dynamic Programming Method with the associated Hamilton-Jacobi-Bellman HJB equations for a particular family of such problems that has been recently studied, namely, the optimal control of heterogeneous systems. Why model heteorogeneity in economics? Economic models have traditionally been built under several simplifying assumptions for a number of reasons, including tractability. Among the reasons, we consider the following three: Considering a single agent to represent the average behaviour of a large number of consumers, for example, greatly simplifies the analysis of an economic system and has enabled the development of a large and coherent body of economic research. As an example, neoclassical growth theory, which has been tremendously influential, considers a representative consumer and a representative firm in place of thousands or millions of separate consumers and firms. Capital homogeneity, which is the lumping together all the forms of capital investment, including human capital and physical capital, is a second simplifying assumption often made in economics. Again the neoclassical growth theory makes this assumption and treats capital investments at different times vintages as identical. This, of course, is hardly realistic, since new vintages typically embody the latest technical improvements and are likely to be significantly more productive. Basically, this evolution is due to two important factors. A major factor is the emerging view that heterogeneity is needed to explain key economic facts. At the same time, the rapid development of computational economicsâ€"especially in the last decadeâ€"makes it feasible to deal with models having heterogeneous agents. Special issues of the reference journal in the field, Journal of Economic Dynamics and Control, have been devoted to this specific area issue 1 in and issue 2 in , suggesting that it is one of the hottest areas in the field of computational economics. An example of results We consider, for example, the vintage capital model. Beginning with the easiest neoclassical growth model the so-called AK model one generalizes it to the case when capital is heterogeneous in the sense that capital is differentiated by its age vintage capital. The basic equation the State Equation in the language of optimal control becomes a differential delay equation. In this case the introduction of heterogeneity allows a more faithful description of the features of this economic system. Indeed, in the graph below Boucekkine et al. Fluctuations of the output is a well known feature that is captured with infinite dimensional optimal control models. Further directions Further work needs to be done with heterogeneity resulting from the spatial and population distribution of economic activity. This heterogeneity is a key feature of contemporary economic systems and a deep study of such models incorporating heterogeneity should both provide more insight into the behavior of such systems and more help for policy makers making decisions. In particular the issues under study are: References Giorgio Fabbri and Fausto Gozzi. Solving optimal growth models with vintage capital: Journal of Economic Theory , no. Vintage capital and the dynamics of the AK model. This entry was posted in Economics by Guest Blogger. Leave a Reply Your email address will not be published.

Chapter 3 : Optimal control - Wikipedia

â€¢ An introduction to mathematical optimal control theory by Bellman's dynamic programming () 10 â€¢ Lev Pontryagin (3 September - 3 May).

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Chapter 5 : Optimal Control HomePage

The main stimuli in the creation of the mathematical theory of optimal control were the discovery of the method of dynamic programming, the explanation of the role of functional analysis in the theory of optimal systems, the discovery of connections between solutions of problems of optimal control and results of the theory of Lyapunov stability.