DOWNLOAD PDF PIECEWISE-SMOOTH DYNAMICAL SYSTEMS

Chapter 1 : May the Piecewise-smooth, Smooth, and Slow-fast Plankton be with You

A piecewise-smooth dynamical system (PWS) is a discrete- or continuous-time dynamical system whose phase space is partitioned in different regions, each associated to a different functional form of the system vector field.

Different types of PWS systems Piecewise-smooth dynamical flows can be classified according to their degree of smoothness, defined as follows. They are typically termed as impacting systems or piecewise-smooth hybrid systems. A classical example is that of an impact oscillator in mechanics see Example 1 below. Systems with DoS equal to 1 have discontinuous vector fields and are usually known as Filippov systems see Example 2 and related article in Scholarpedia [Filippov,][Leine et al. Piecewise-smooth dynamical systems can exhibit most of the bifurcations also exhibited by smooth systems such as period-doublings, saddle-nodes , homoclinic tangencies, etc. In addition to these, they can also exhibit some novel bifurcation phenomena which are unique to piecewise smooth systems or discontinuity-induced bifurcations DIBs [diBernardo et al. A DIB in this sense is any transition observed in the system under investigation which can be explained in terms of interactions between its invariant sets and its switching manifolds in phase space. Thus, DIBs include interactions of fixed points , equilibria and limit cycles with the system switching manifolds. Invariant tori and chaotic sets can also undergo DIBs but these have been little investigated in the scientific literature so far. Let us list some of the most commonly occurring types of codimension-one DIBs see Figure 2. Border collisions of maps [Fig. Remarkably, the unfolding may be quite complex. See Example 4 below. Boundary equilibrium bifurcations [Fig. There are thus possibilities where the equilibrium lies precisely on the boundary between a sliding or sticking region and a pseudo-equilibrium turns into a regular equilibrium either under direct parameter variation or in a fold-like transition where both exist for the same sign of the perturbing parameter. There is also the possibility that a limit cycle may be spawned under parameter perturbation of the boundary equilibrium, in a Hopf-like transition. Grazing bifurcations of limit cycles [Fig. One of the most commonly found DIBs in applications is caused by a limit cycle of a flow becoming tangent to i. However, this is not necessarily the case, as one has to analyze carefully what happens to the flow in the neighborhood of the grazing point. In fact, one can derive an associated map the, so-called, discontinuity map. Sliding and sticking bifurcations [Fig. There are several ways that an invariant set such as a limit cycle can do something structurally unstable with respect to the boundary of a sliding region in a Filippov system. Another possibility for a codimension-one event in a flow is where an invariant set e. Thus we think of this body as a single particle in space. It can be shown that the most fundamental mechanism organizing the observed complex behaviour is the grazing bifurcation of limit cycles causing the appearance and disappearance of various attractors. A typical grazing bifurcation phenomenon is shown in Figure 4 a. It can be shown that locally to the grazing event, the dynamics of the system can be described by a discrete-time mapping characterized by a square-root singularity. Hence, a ball of initial conditions is infinitely stretched in one direction as the impact velocity tends towards zero at grazing. This can cause dramatic changes in the system dynamics as, for example, the sudden transition from a periodic to a chaotic attractor shown in Figure 4 b. Illustration in three dimensions of the four codimension-one bifurcation scenarios involving collision of a segment of trajectory with the boundary of the sliding region: A simple example of Filippov dynamics occurs in feedback relay control systems. The idea of using a switching action or relay has been widely employed in control engineering since the s. Indeed, relay control has been used, for instance, in pulsed servomechanisms, tuning controllers in the process industry. More generally, relay systems play an important role in the theory of variable structure controllers [Utkin,], of hybrid systems [Van der Schaft et al. Although systems with a relay feedback have been studied for a long time for example, in the work of Andronov and Flugge-Lotz from the s and s , the dynamics of these systems is not fully understood. It has been shown, for example, that even low-order relay feedback systems can exhibit more complex self-oscillations either periodic or chaotic , which include segments of sliding motion [di Bernardo et al. Examples of engineering control systems with relay elements featuring chaotic behavior as well as quasi-periodic solutions are discussed in [Cook,]. Here we consider a simple class of model problems corresponding to single-input--single-output, linear, time-invariant relay control systems with unit negative feedback of the output variable. Such problems can be written in the general form: Furthermore, it is assumed that the system matrices are given in observer canonical form; i. In particular, it has been shown that four main types of sliding bifurcations of limit cycles can occur in Filippov systems. Figure 5 depicts schematically each of these cases. The crossing-sliding bifurcation of a periodic solution of a three-dimensional relay feedback system of the form 1 is shown in Figure 6. For more information see the related Scholarpedia article on Filippov systems. Schematic diagram of a simple DC--DC buck converter circuit. In the past, this was done by converting the DC voltage to an AC one, passing this through a transformer, and then transforming the resulting AC voltage back to a DC one. This procedure results in significant energy loss and rather bulky devices. To convert between voltages with domestic electronic devices, such as laptop computers, something more compact and with less energy loss is needed. The DC--DC converters frequently employed use electronic switches to convert from one DC voltage to another, with negligible energy loss. Significantly, such mechanisms can be implemented using small solid state devices. The use of switches means that DC--DC converters represent inherently non-smooth dynamical systems, which when driven beyond their designed operating limits can give rise to complex dynamics of the form studied in this book. DC--DC converter bifurcation diagram, obtained by direct numerical simulation. The corresponding mathematical model is a PWS system with degree of smoothness equal to unity Filippov. DC--DC converters have been shown to exhibit complex behaviour including several types of bifurcations and chaos see bifurcation diagram in Figure 8. The organizing DIB, causing the sudden jump to a large-amplitude chaotic evolution in this case is a corner-collision bifurcations causing the sudden transition from periodic to chaotic attractors for more details on this diagram see [Banerjee, Verghese] and references therein. In particular, Figure 9 a shows the transition from a one-periodic to a four-periodic attractor while Figure 9 b the transition from a one-periodic to a chaotic attractor observed for slightly different parameter values. The most notable cascade is the period-adding shown in Figure 10 where periodic orbits of increasing periodicity alternate with bands of chaotic evolution. This is a typical phenomenon often observed in PWS systems which is clearly different from the period-doubling cascade to chaos often observed in smooth dynamical systems. Phase portraits corresponding to a boundary-equilibrium bifurcation of the piecewise-smooth flow. Verghese Nonlinear Phenomena in Power Electronics: Kowalczyk Piecewise-smooth dynamical systems: Theory and Applications, Springer doi: Homer Local Analysis of C-bifurcations in n-dimensional piecewise smooth dynamical systems. Chaos, Solitons and Fractals , International Journal of Bifurcations and Chaos, 11 4: Cook Simple feedback systems with chaotic behaviour. Feigin Doubling of the oscillation period with C-bifurcations in piecewise continuous systems, Prikladnaya Matematika i Mekhanika Feigin The increasingly complex structure of the bifurcation tree of a piecewise-smooth system, J. Budd Analytical and experimental investigation of an impact oscillator, Proc. Van der Schaft and J. Shea-Brown Periodic orbit. James Murdock Unfoldings.

Chapter 2 : Piecewise-smooth Dynamical Systems : Mario Di Bernardo :

Traditional analysis of dynamical systems has restricted its attention to smooth problems, but it has become increasingly clear that there are distinctive phenomena unique to discontinuous systems that can be analyzed mathematically but which fall outside the usual methodology for smooth dynamical.

Chapter 3 : Piecewise smooth dynamical systems - Scholarpedia

*Piecewise-smooth Dynamical Systems: Theory and Applications (Applied Mathematical Sciences) [Mario Bernardo, Chris Budd, Alan Richard Champneys, Piotr Kowalczyk] on www.nxgvision.com *FREE* shipping on qualifying offers. This book presents a coherent framework for understanding the dynamics of piecewise-smooth and hybrid systems.*

Chapter 4 : quantum mechanics - Examples of piecewise smooth dynamical systems - Physics Stack Exch

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