DOWNLOAD PDF STOCHASTIC DIFFERENTIAL EQUATIONS AN INTRODUCTION WITH APPLICATION

Chapter 1 : Stochastic Differential Equations : Bernt Ã–ksendal :

This book gives an introduction to the basic theory of stochastic calculus and its applications. Examples are given throughout the text, in order to motivate and illustrate the theory and show its importance for many applications in e.g. economics, biology and physics.

This work was followed upon by Langevin. Terminology[edit] The most common form of SDEs in the literature is an ordinary differential equation with the right hand side perturbed by a term dependent on a white noise variable. In most cases, SDEs are understood as continuous time limit of the corresponding stochastic difference equations. This understanding of SDEs is ambiguous and must be complemented by an "interpretation". An alternative view on SDEs is the stochastic flow of diffeomorphisms. This understanding is unambiguous and corresponds to the Stratonovich version of the continuous time limit of stochastic difference equations. Associated with SDEs is the Smoluchowski equation or the Fokkerâ€”Planck equation , an equation describing the time evolution of probability distribution functions. The generalization of the Fokkerâ€”Planck evolution to temporal evolution of differential forms is provided by the concept of stochastic evolution operator. In physical science, there is an ambiguity in the usage of the term "Langevin SDEs". While Langevin SDEs can be of a more general form , this term typically refers to a narrow class of SDEs with gradient flow vector fields. From the physical point of view, however, this class of SDEs is not very interesting because it never exhibits spontaneous breakdown of topological supersymmetry, i. Stochastic calculus[edit] Brownian motion or the Wiener process was discovered to be exceptionally complex mathematically. The Wiener process is almost surely nowhere differentiable; thus, it requires its own rules of calculus. Each of the two has advantages and disadvantages, and newcomers are often confused whether the one is more appropriate than the other in a given situation. Still, one must be careful which calculus to use when the SDE is initially written down. Numerical solutions[edit] Numerical solution of stochastic differential equations and especially stochastic partial differential equations is a young field relatively speaking. Almost all algorithms that are used for the solution of ordinary differential equations will work very poorly for SDEs, having very poor numerical convergence. Use in physics[edit] In physics, SDEs have widest applicability ranging from molecular dynamics to neurodynamics and to the dynamics of astrophysical objects. More specifically, SDEs describe all dynamical systems, in which quantum effects are either unimportant or can be taken into account as perturbations. SDEs can be viewed as a generalization of the dynamical systems theory to models with noise. This is an important generalization because real systems cannot be completely isolated from their environments and for this reason always experience external stochastic influence. There are standard techniques for transforming higher-order equations into several coupled first-order equations by introducing new unknowns. Therefore, the following is the most general class of SDEs:

Chapter 2 : Geometric Brownian motion - Wikipedia

It is a serious introduction that starts with fundamental measure-theoretic concepts and ends, coincidentally, with the Black-Scholes formula as one of several examples of applications. This is the best single resource for learning the stochastic calculus .".

So far we had a function, and then we differentiated to get an equation of this type. And we have to go backwards, want to find the stochastic process that satisfies this equation. So goal is to find a stochastic process, xt, satisfying this equation. In other words, we want x of t to be the integral of mu ds plus sigma [? And so these type of equations are called differential equations, I hope you already know that, also for PDE, partial differential equations. Still, a very important result that you should first note before trying to solve any of the differential equation is that, as long as mu and sigma are reasonable functions, there does exist a solution. So we have the same correspondence with this PDE. So the same principle holds in stochastic world. Now, let me state it formally. This stochastic equation, star, has a solution that is unique, of course with [? Just check, yes-- as long as mu and sigma are reasonable. One way it can be reasonable, if it satisfies this conditions. These are very technical conditions. But at least let me parse to you what they are. They say, if you fix a time coordinate and you change x, you look at the difference between the values of mu when you change the second variable and the sigma. Then the change in your function is bounded by the distance between the two points by some constant k. So mu and sigma cannot change too much when you change your space variable a little bit. It can only change up to the distance of how much you change the coordinate. Second condition says, when you go to your x, very similar condition. Essentially it says it cannot blow up too fast, the whole thing. This one is something about the difference between two values. This one is about how it expands as your space variable grows. These are technical conditions. And many cases, they will hold. But you do expect to have a solution of some form. Here is one of the few stochastic differential equations that can be solved. This one can be solved. And you already know what x is. I will show you an approach, which can solve some differential equations, some SDEs. If you want this to happen, then d of x of t is-- OK. And then these two have to match. That has to be equal to that. So we know that del f over del t, mu of x of t is equal to mu of x. So we assumed that this is a solution then differentiated that. You get this equation. If you look at that, that tells you that f is an exponential function in the x variable. And then we have a constant term, plus some a times a function of t. And when you fix a t and change x, it has to look like an exponential function. It has to be in this form, just by the second equation. Now, go back to this equation. What you get is partial f over partial t is now a times g prime of t [? That one in second to last line, yeah. So why is it minus mu there at the end? In other words, a of t prime of t is mu minus square. OK, and then what we got is original function, f of t of x is e to the sigma x plus mu minus 1 over 2 sigma square t plus some constant. And that constant can be chosen because we have the initial condition, x of 0 equals 0. That means if t is equal to 0, f 0, 0 is equal to e to the c. That has to be x0. In different words, this is just x0 times e to the sigma x plus mu minus 1 over 2 sigma square of t. Just as we expected, we got this. The sum of two stochastic differential equations can be solved by analyzing it. Just looking at it, and, you know, OK, it has to be exponential function. Because, in fact, what we did is we kind of already knew the answer and are fitting into that answer. Let me give you one more example where we do have an explicit solution. Second equation is called this. So previously, our main drift term also was proportional to the current value. And the error was also dependent on the current value or is proportional to the current value. But here now the drift term is something like an exponential minus the exponential. But the error term is just some noise. Irrelevant of what the value is, this has the same variance as the error. And this is used to model a mean reverting stochastic process. For alpha greater than 0, notice that if x deviates from 0, this gives a force that drives the stochastic process back to 0. This negatively proportional to your current value. So yeah, this is used to model some mean reverting stochastic processes. And they first used it to study the behavior of gases. Anyway, so this is another thing that can be solved by doing similar analysis. But if you try the same method, it will fail. So as a test function, your guess, initial guess, will be-- or a0 is equal to 1. And then try this, try that, and eventually something might succeed. And it was the same when, if you remember how we solved ordinary differential equations or partial differential equations, most of the time there is no good guess. Differentiate it, and let me go slow. So we have a prime of t. By chain rule, a prime of t and that value, that part will be equal to xt over at. This is chain rule. So I differentiate that to get a prime t. That stays just as it was. But that can be rewritten as xt divided by at. And then plus at times the differential of that one. And that is just b t dB t. But when you have a given stochastic process written already in this integral form, if we remember the definition of an integral, at least how I defined it, is that it was an inverse operation of a differential. So when you differentiate this, you just get that term. This can be confusing. But think about it. Now, we laid it out and just compare. So minus alpha of x of t is equal to a prime t over at xt. And your second term, sigma dBT is equal to at times bt. But these two cancel. And with these, at has to be e to the minus alpha t. Now, plug it in here. So plug it back in. So if this is a variance, expectation is 0. And that is kind of hinted by this fact, the mean reversion. So if you start from some value, at least the drift term will go to 0 quite quickly. And then the important term will be the noise term or the variance term. Because most of the times, it will be useless. What if such method fails? And it will fail most of the time. The finite difference method, you probably already saw it, if you took a differential equation course. But let me review it.

DOWNLOAD PDF STOCHASTIC DIFFERENTIAL EQUATIONS AN INTRODUCTION WITH APPLICATION

Chapter 3 : An Introduction To Stochastic Differential Equations | Download PDF EPUB eBook

Stochastic Differential Equations has 54 ratings and 6 reviews. Dmitri said: This book is the 'principal' text used by, ahem, everyone in graduate course.

Chapter 11 Section I thank them all for their contributions to the improvement of the book. I am very grateful for her help and for her patience with me and all my revisions, new versions and revised revisions. In addition solutions and extra hints to some of the exercises are now included. Moreover, the proof and the discussion of the Girsanov theorem have been changed in order to make it more easy to apply, e. And the presentation in general has been corrected and revised throughout the text, in order to make the book better and more useful. I am grateful to them all for their help. The purpose of these exercises is to help the reader to get a better understanding of the text. Some of the exercises are quite routine, intended to illustrate the results, while other exercises are harder and more challenging and some serve to extend the theory. I have also continued the effort to correct misprints and errors and to improve the presentation. I have benefitted from valuable comments and suggestions from Mark H. A quite noticeable non-mathematical improvement is that the book is now typed in TE X. Tove Lieberg did a great typing job as usual and I am very grateful to her for her effort and infinite patience. Chapter VII treats only those basic properties of diffusions that are needed for the applications in the last 3 chapters. Hopefully this change will make the book more flexible for the different purposes. I have also made an effort to improve the presentation at some points and I have corrected the misprints and errors that I knew about, hopefully without introducing new ones. I am grateful for the responses that I have received on the book and in particular I wish to thank Henrik Martens for his helpful comments. Tove Lieberg has impressed me with her unique combination of typing accuracy and speed. I wish to thank her for her help and patience, together with Dina Haraldsson and Tone Rasmussen who sometimes assisted on the typing. No previous knowledge about the subject was assumed, but the presentation is based on some background in measure theory. There are several reasons why one should learn more about stochastic differential equations: They have a wide range of applications outside mathematics, there are many fruitful connections to other mathematical disciplines and the subject has a rapidly developing life of its own as a fascinating research field with many interesting unanswered questions. Unfortunately most of the literature about stochastic differential equations seems to place so much emphasis on rigor and completeness that it scares many nonexperts away. These notes are an attempt to approach the subject from the nonexpert point of view: Not knowing anything except rumours, maybe about a subject to start with, what would I like to know first of all? My answer would be: I would not be so interested in the proof of the most general case, but rather in an easier proof of a special case, which may give just as much of the basic idea in the argument. And I would be willing to believe some basic results without proof at first stage, anyway in order to have time for some more basic applications. These notes reflect this point of view. Such an approach enables us to reach the highlights of the theory quicker and easier. Thus it is hoped that these notes may contribute to fill a gap in the existing literature. The course is meant to be an appetizer. If it succeeds in awaking further interest, the reader will have a large selection of excellent literature available for the study of the whole story. Some of this literature is listed at the back. In the introduction we state 6 problems where stochastic differential equations play an essential role in the solution. In Chapter II we introduce the basic mathematical notions needed for the mathematical model of some of these problems, leading to the concept of Ito integrals in Chapter III. In Chapter IV we develop the stochastic calculus the Ito formula and in Chap- XVI ter V we use this to solve some stochastic differential equations, including the first two problems in the introduction. In Chapter VI we present a solution of the linear filtering problem of which problem 3 is an example , using the stochastic calculus. Problem 4 is the Dirichlet problem. Problem 5 is an optimal stopping problem. In Chapter IX we represent the state of a game at time t by an Ito diffusion and solve the corresponding optimal stopping problem. The solution involves potential theoretic notions, such as the generalized harmonic extension provided by the solution of the Dirichlet problem in Chapter VIII. Problem 6 is a stochastic version of F. In Chapter X we formulate the general stochastic control problem in terms of stochastic differential equations, and we apply the results of Chapters VII and VIII to show that the problem can be reduced to solving the deterministic Hamilton-Jacobi-Bellman equation. As an illustration we solve a problem about optimal portfolio selection. After the course was first given in Edinburgh in , revised and expanded versions were presented at Agder College, Kristiansand and University of Oslo. This fruitful combination has created a broad variety of valuable comments, for which I am very grateful. I particularly wish to express my gratitude to K. Davie for many useful discussions. And I am greatly indebted to Ingrid Skram, Agder College and Inger Prestbakken, University of Oslo for their excellent typing â€” and their patience with the innumerable changes in the manuscript during these two years. Table of Contents 1. Other Topics in Diffusion Theory. Applications to Boundary Value Problems. Application to Optimal Stopping. Application to Stochastic Control. Application to Mathematical Finance. XIX Appendix A: Uniform Integrability and Martingale Convergence. Introduction To convince the reader that stochastic differential equations is an important subject let us mention some situations where such equations appear and can be used: The function r t is assumed to be nonrandom. How do we solve 1. Introduction How do we solve 1. More generally, the equation we obtain by allowing randomness in the coefficients of a differential equation is called a stochastic differential equation. This will be made more precise later. It is clear that any solution of a stochastic differential equation must involve some randomness, i. However, due to inaccuracies in our measurements we do not really measure Q s but a disturbed version of it: The filtering problem is: What is the best estimate of Q t satisfying 1. Almost immediately the discovery found applications in aerospace engineering Ranger, Mariner, Apollo etc. For the Kalman-Bucy filter â€” as the whole subject of stochastic differential equations â€” involves advanced, interesting and first class mathematics. The most celebrated example is the stochastic solution of the Dirichlet problem: In Kakutani proved that the solution could be expressed in terms of Brownian motion which will be constructed in Chapter 2: It turned out that this was just the tip of an iceberg: For a large class of semielliptic second order partial differential equations the corresponding Dirichlet boundary value problem can be solved using a stochastic process which is a solution of an associated stochastic differential equation. Suppose a person has an asset or resource e. The price Xt at time t of her asset on the open market varies according to a stochastic differential equation of the same type as in Problem 1: At what time should she decide to sell? We assume that she knows the behaviour of Xs up to the present time t, but because of the noise in the system she can of course never be sure at the time of the sale if her choice of time will turn out to be the best. So what we are searching for is a stopping strategy that gives the best result in the long run, i. This is an optimal stopping problem. It turns out that the solution can be expressed in terms of the solution of a corresponding boundary value problem Problem 4 , except that the boundary is unknown free as well and this is compensated by a double set of boundary conditions. It can also be expressed in terms of a set of variational inequalities. Suppose that a person has two possible investments: Such a right is called a European call option. How much should the person be willing to pay for such an option? This problem was solved when Fischer Black and Myron Scholes used stochastic analysis and an equlibrium argument to compute a theoretical value for the price, the now famous Black and Scholes option price formula. This theoretical value agreed well with the prices that had already been established as an equilibrium price on the free market. Thus it represented a triumph for mathematical modelling in finance. It has become an indispensable tool in the trading of options and other financial derivatives. Fischer Black died in We will return to these problems in later chapters, after having developed the necessary mathematical machinery. We solve Problem 1 and Problem 2 in Chapter 5. Problems involving filtering Problem 3 are treated in Chapter 6, the generalized Dirichlet problem Problem 4 in Chapter 9. Problem 5 is solved in Chapter 10 while stochastic control problems Problem 6 are discussed in Chapter Finally we discuss applications to mathematical finance in Chapter Some Mathematical Preliminaries 2. In short, here is a first list of the notions that need a mathematical interpretation: In this chapter we will discuss 1 â€” 3 briefly. In Chapter 6 we will consider 4 â€” 5.

DOWNLOAD PDF STOCHASTIC DIFFERENTIAL EQUATIONS AN INTRODUCTION WITH APPLICATION

Chapter 4 : Stochastic differential equation - Wikipedia

To convince the reader that stochastic differential equations is an important subject let us mention some situations where such equations appear and can be used.

Chapter 5 : Stochastic Differential Equations: An Introduction with Applications by Bernt Ã˜ksendal

A highly readable introduction to stochastic integration and stochastic differential equations, this book combines developments of the basic theory with applications. It is written in a style suitable for the text of a graduate course in stochastic calculus, following a course in probability.

Chapter 6 : Stochastic Differential Equations - Download Free EBooks

mathematics and statistics, Stochastic Differential Equations: An Introduction with Applications in Population Dynamics Modeling is an excellent fit for advanced under- graduates and beginning graduate students, as well as practitioners who need a gentle.

Chapter 7 : Stochastic Differential Equations: An Introduction with Applications - PDF Free Download

Stochastic Differential Equations: An Introduction with Applications / Edition 6 This book gives an introduction to the basic theory of stochastic calculus and its applications. Examples are given throughout the text, in order to motivate and illustrate the theory and show its importance for many applications in e.g. economics, biology and physics.

Chapter 8 : Stochastic differential equations : an introduction with applications in SearchWorks catalog

Bernt Oksendal is a proven Springer author. His book, Stochastic Differential Equations: An Introduction with Applications, has sold over 10, copies and has been translated into Japanese and Chinese.

Chapter 9 : Stochastic Differential Equations | Course Web Pages

Stochastic dïï¬€erential equations is usually, and justly, regarded as a graduate level subject. A really careful treatment assumes the students' familiarity with probability.