

## Chapter 1 : High School Statistics | Khan Academy

*Probability and statistics, the branches of mathematics concerned with the laws governing random events, including the collection, analysis, interpretation, and display of numerical data. Probability has its origin in the study of gambling and insurance in the 17th century, and it is now an indispensable tool of both social and natural sciences.*

A very comprehensive collection of links, created by Henk Wolthuis, not only to actuarial science and demography, but to statistics as well. To help place individuals I have used modern terms for occupation e. For the earlier figures these terms are anachronistic but, I hope, not too misleading. I have not given nationality as people move and states come and go. MacTutor has plenty of geographical information. This was the age of the Scientific Revolution and the biggest names, Galileo Materials and Todhunter ch. Hacking Chapters discusses thinking before Pascal. Gregory King was an important figure in the next generation. However the economic line fizzled out. Actuarial questions and friendship with the actuary G. Lidstone stimulated the Edinburgh mathematicians, E. Aitken MGPP , to contribute to statistics and numerical analysis. The C17 work is discussed by Hacking Chapter 13 , Annuities. There are historical articles in the Encyclopedia of Actuarial Science. Classics are reprinted in History of Actuarial Science. There is a nice review of the early literature in the catalogue of the Equitable Life Archive. The Berlin and St. Petersburg academies were formed a bit later. The Royal Society was a forerunner of the modern scientific society, while the continental academies were more like research institutes. For the period generally see Todhunter ch. Top Blaise Pascal Mathematician and philosopher. Pascal was educated at home by his father, himself a considerable mathematician. The origins of probability are usually found in the correspondence between Pascal and Fermat where they treated several problems associated with games of chance. The letters were not published but their contents were known in Parisian scientific circles. The last chapter of the Port-Royal Logic pp. Edwards on the triangle and Todhunter ch. As a youth Huygens was expected to become a diplomat but instead he became a gentleman scientist, making important contributions to mathematics, physics and astronomy. He was educated at the University.

## Chapter 2 : Lies, Damned Lies, Statistics, and Probability of Abiogenesis Calculations

*Probability is distinguished from statistics; see history of statistics. While statistics deals with data and inferences from it, (stochastic) probability deals with the stochastic (random) processes which lie behind data or outcomes.*

Since participation in games and gambling is as old as mankind, it seems as if the idea of probability should be almost as old. But, the realization that one could predict an outcome to a certain degree of accuracy was unconceivable until the sixteenth and seventeenth centuries. This idea that one could determine the chance of a future event was prompted from the necessity to achieve a predictable balance between risks taken and the potential for gain. In order to make a profit, underwriters were in need of dependable guidelines by which a profit could be expected, while the gambler was interested in predicting the possibility of gain. The first recorded evidence of probability theory can be found as early as in the work of Cardan. In Cardan wrote a manuscript in which he addressed the probability of certain outcomes in rolls of dice, the problem of points, and presented a crude definition of probability. Had this manuscript not been lost, Cardan would have certainly been accredited with the onset of probability theory. However, the manuscript was not discovered until and printed in , leaving the door open for independent discovery. The onset of probability as a useful science is primarily attributed to Blaise Pascal and Pierre de Fermat While contemplating a gambling problem posed by Chevalier de Mere in , Blaise Pascal and Pierre de Fermat laid the fundamental groundwork of probability theory, and are thereby accredited the fathers of probability. The question posed was pertaining to the number of turns required to ensure obtaining a six in the roll of two dice. The correspondence between Pascal and Fermat concerning this and the problem of points led to the beginning of the new concepts of probability and expectation. In the seventeenth century, a shopkeeper, John Graunt , set out to predict mortality rates by categorizing births and deaths. In the London Life Table, Graunt made a noteworthy attempt to predict the number of survivors out of one hundred through increments of ten years. This work along with his earlier paper Natural and Political Observations Made upon the Bills of Mortality , the first known paper to use data in order to draw statistical inferences, gained him access into the Royal Society of London. In this text, Huygens presented the idea of mathematical expectation. This text was unrivaled until James Bernoulli wrote *Ars Conjectandi* , which was published eight years after his death. The *Theorie Analytique des Probabilites* outlined the evolution of probability theory, providing extensive explanations of the results obtained. In this book Laplace presented the definition of probability, which we still use today, and the fundamental theorems of addition and multiplication of probabilities along with several problems applying the Bernoulli process. The first major accomplishment in the development of probability theory was the realization that one could actually predict to a certain degree of accuracy events which were yet to come. It was the initial work of Pascal, Fermat, Graunt, Bernoulli, DeMoivre, and Laplace that set probability theory, and then statistics, on its way to becoming the valuable inferential science that it is today.

**Chapter 3 : History of Mathematics: History of Probability and Statistics**

*The history of statistics in the modern sense dates from the mid 19th century, with the term statistics itself coined in German, although there have been changes to the interpretation of the word over time.*

Acknowledgements Introduction Every so often, someone comes up with the statement "the formation of any enzyme by chance is nearly impossible, therefore abiogenesis is impossible". These people, including Fred, have committed one or more of the following errors. Glossary An enzyme or ribozyme that synthesizes peptides. An enzyme or ribozyme that adds a monomer to a polymer, or links two shorter polymers together. Any single subunit of a polymer. An amino acid is a monomer of a peptide or protein, a nucleotide is a monomer of an oligonucleotide or polynucleotide. Adenine, Guanine, Cytosine and Uracil. These are the monomers that make up oligo- or polynucleotides such as RNA. A short polymer of nucleotide subunits. A enzyme or ribozyme that makes a polymer out of monomers. A biological catalyst made from RNA. A molecule which can make an identical or near-identical copy of itself from smaller subunits. At least four self-replicators are known. This is not the abiogenesis theory at all. I will try and walk people through these various errors, and show why it is not possible to do a "probability of abiogenesis" calculation in any meaningful way. This is then cranked up by adding on the probabilities of generating or so similar enzymes until a figure is reached that is so huge that merely contemplating it causes your brain to dribble out your ears. This gives the impression that the formation of even the smallest organism seems totally impossible. However, this is completely incorrect. Firstly, the formation of biological polymers from monomers is a function of the laws of chemistry and biochemistry, and these are decidedly not random. Secondly, the entire premise is incorrect to start off with, because in modern abiogenesis theories the first "living things" would be much simpler, not even a protobacteria, or a preprotobacteria what Oparin called a protobiont [ 8 ] and Woese calls a progenote [ 4 ] , but one or more simple molecules probably not more than subunits long. These simple molecules then slowly evolved into more cooperative self-replicating systems, then finally into simple organisms [ 2 , 5 , 10 , 15, 28 ]. An illustration comparing a hypothetical protobiont and a modern bacteria is given below. The first "living things" could have been a single self replicating molecule, similar to the "self-replicating" peptide from the Ghadiri group [ 7 , 17 ], or the self replicating hexanucleotide [ 10 ], or possibly an RNA polymerase that acts on itself [ 12 ]. Another view is the first self-replicators were groups of catalysts, either protein enzymes or RNA ribozymes, that regenerated themselves as a catalytic cycle [ 3 , 5 , 15 , 26 , 28 ]. An example is the SunY three subunit self-replicator [ 24 ]. These catalytic cycles could be limited in a small pond or lagoon, or be a catalytic complex adsorbed to either clay or lipid material on clay. These two models are not mutually exclusive. The Ghadiri peptide can mutate and form catalytic cycles [ 9 ]. Just to hammer this home, here is a simple comparison of the theory criticised by creationists, and the actual theory of abiogenesis. Each step is associated with a small increase in organisation and complexity, and the chemicals slowly climb towards organism-hood, rather than making one big leap [ 4 , 10 , 15 , 28 ]. Where the creationist idea that modern organisms form spontaneously comes from is not certain. Because there are some fundamental problems in statistics and biochemistry that turn up in these mistaken "refutations". The myth of the "life sequence" Another claim often heard is that there is a "life sequence" of proteins, and that the amino acid sequences of these proteins cannot be changed, for organisms to be alive. This, however, is nonsense. The protein claim seems to come from the protein coding genome of *Mycobacterium genitalium*, which has the smallest genome currently known of any modern organism [ 20 ]. However, inspection of the genome suggests that this could be reduced further to a minimal gene set of proteins [ 20 ]. Note again that this is a modern organism. As to the claim that the sequences of proteins cannot be changed, again this is nonsense. There are in most proteins regions where almost any amino acid can be substituted, and other regions where conservative substitutions where charged amino acids can be swapped with other charged amino acids, neutral for other neutral amino acids and hydrophobic amino acids for other hydrophobic amino acids can be made. In fact it is possible to substitute structurally non-identical bacterial proteins for yeast proteins, and worm proteins for human proteins, and the organisms live quite happily. The "life sequence" is a myth. This

certainly is not the way peptides formed on the early Earth, but it will be instructive. I will use as an example the "self-replicating" peptide from the Ghadiri group mentioned above [ 7 ]. I could use other examples, such as the hexanucleotide self-replicator [ 10 ], the SunY self-replicator [ 24 ] or the RNA polymerase described by the Eckland group [ 12 ], but for historical continuity with creationist claims a small peptide is ideal. It is also of a size and composition that is ideally suited to be formed by abiotic peptide synthesis. The fact that it is a self replicator is an added irony. This is much, much more probable than the 1 in 2. When someone tells us that some event has a one in a million chance of occurring, many of us expect that one million trials must be undergone before the said event turns up, but this is wrong. Here is a experiment you can do yourself: How many times would you think you had to repeat this procedure trial before you get 4 heads in a row? Even at 1 chance in 4. But there is more. Even with the argument above you could get it on your very first trial most people would say "surely it would still take more time than the Earth existed to make this replicator by random methods". In fact there would be billions of simultaneous trials as the billions of building block molecules interacted in the oceans, or on the thousands of kilometers of shorelines that could provide catalytic surfaces or templates [ 2 , 15 ]. Say it takes a minute to toss the coins 4 times; to generate HHHH would take on average 8 minutes. Now get 16 friends, each with a coin, to all flip the coin simultaneously 4 times; the average time to generate HHHH is now 1 minute. This would take half an hour on average, but go out and recruit 64 people, and you can flip it in a minute. If you want to flip a sequence with a chance of 1 in a billion, just recruit the population of China to flip coins for you, you will have that sequence in no time flat. So, if on our prebiotic earth we have a billion peptides growing simultaneously, that reduces the time taken to generate our replicator significantly. Okay, you are looking at that number again, 1 chance in 4. Yes, one kilogram of the amino acid arginine has 2. If you took a semi-trailer load of each amino acid and dumped it into a medium size lake, you would have enough molecules to generate our particular replicator in a few tens of years, given that you can make 55 amino acid long proteins in 1 to 2 weeks [ 14 , 16 ]. So how does this shape up with the prebiotic Earth? On the early Earth it is likely that the ocean had a volume of  $1 \times 10^{21}$  litres. Given an amino acid concentration of  $1 \times 10^{-6}$  M a moderately dilute soup, see Chyba and Sagan [ 23 ], then there are roughly  $1 \times 10^{15}$  potential starting chains, so that a fair number of efficient peptide ligases about  $1 \times 10^6$  could be produced in a year, let alone a million years. The synthesis of primitive self-replicators could happen relatively rapidly, even given a probability of 1 chance in 4. Assume that it takes a week to generate a sequence [ 14 , 16 ]. Then the Ghadiri ligase could be generated in one week, and any cytochrome C sequence could be generated in a bit over a million years along with about half of all possible peptide sequences, a large proportion of which will be functional proteins of some sort. Although I have used the Ghadiri ligase as an example, as I mentioned above the same calculations can be performed for the SunY self replicator, or the Eckland RNA polymerase. I leave this as an exercise for the reader, but the general conclusion you can make scads of the things in a short time is the same for these oligonucleotides. Search spaces, or how many needles in the haystack? However, an analysis by Eckland suggests that in the sequence space of nucleotide long RNA sequences, a staggering  $2 \times 10^{15}$ . Not bad for a compound previously thought to be only structural. Going back to our primitive ocean of  $1 \times 10^{21}$  litres and assuming a nucleotide concentration of  $1 \times 10^{-6}$  M [ 23 ], then there are roughly  $1 \times 10^{15}$  potential nucleotide chains, so that a fair number of efficient RNA ligases about  $1 \times 10^6$  could be produced in a year, let alone a million years. Similar considerations apply for ribosomal acyl transferases about 1 in every sequences , and ribozymal nucleotide synthesis [ 1 , 6 , 13 ]. Similarly, of the  $1 \times 10^6$  possible unit proteins, 3. So, even with more realistic if somewhat mind beggaring figures, random assemblage of amino acids into "life-supporting" systems whether you go for protein enzyme based hypercycles [ 10 ], RNA world systems [ 18 ], or RNA ribozyme-protein enzyme coevolution [ 11 , 25 ] would seem to be entirely feasible, even with pessimistic figures for the original monomer concentrations [ 23 ] and synthesis times. Furthermore, this argument is often buttressed with statistical and biological fallacies. For the replicating polymers to hypercycle transition, the probability may well be 1. J Theor Biol,

**Chapter 4 : History of Statistics and Probability by Lydia Young on Prezi**

*Studies in the history of statistics and probability: a series of papers.* Griffin, London, Porter, Theodore M. *The rise of statistical thinking*, Princeton University Press, Princeton, Stigler, Stephen M. *The History of Statistics: the measurement of uncertainty before*

Probability density plots for the Laplace distribution. Pierre-Simon Laplace made the first attempt to deduce a rule for the combination of observations from the principles of the theory of probabilities. He represented the law of probability of errors by a curve and deduced a formula for the mean of three observations. Laplace in noted that the frequency of an error could be expressed as an exponential function of its magnitude once its sign was disregarded. Lagrange proposed a parabolic distribution of errors in Laplace in published his second law of errors wherein he noted that the frequency of an error was proportional to the exponential of the square of its magnitude. This was subsequently rediscovered by Gauss possibly in and is now best known as the normal distribution which is of central importance in statistics. Peirce in who was studying measurement errors when an object was dropped onto a wooden base. Lagrange also suggested in two other distributions for errors - a raised cosine distribution and a logarithmic distribution. Laplace gave a formula for the law of facility of error a term due to Joseph Louis Lagrange , , but one which led to unmanageable equations. Daniel Bernoulli introduced the principle of the maximum product of the probabilities of a system of concurrent errors. In William Playfair introduced the idea of graphical representation into statistics. He invented the line chart , bar chart and histogram and incorporated them into his works on economics , the Commercial and Political Atlas. These latter charts came to general attention when he published examples in his Statistical Breviary in In Laplace estimated the population of France to be 28,, The census data of these communities showed that they had 2,, persons and that the number of births were 71, Assuming that these samples were representative of France, Laplace produced his estimate for the entire population. Carl Friedrich Gauss , mathematician who developed the method of least squares in The method of least squares , which was used to minimize errors in data measurement , was published independently by Adrien-Marie Legendre , Robert Adrain , and Carl Friedrich Gauss Gauss had used the method in his famous prediction of the location of the dwarf planet Ceres. The observations that Gauss based his calculations on were made by the Italian monk Piazzi. The term probable error der wahrscheinliche Fehler - the median deviation from the mean - was introduced in by the German astronomer Frederik Wilhelm Bessel. Other contributors to the theory of errors were Ellis , De Morgan , Glaisher , and Giovanni Schiaparelli In the 19th century authors on statistical theory included Laplace, S. Gustav Theodor Fechner used the median Centralwerth in sociological and psychological phenomena. Francis Galton used the English term median for the first time in having earlier used the terms middle-most value in and the medium in The only data sets available to him that he was able to show were normally distributed were birth rates. Development of modern statistics[ edit ] Although the origins of statistical theory lie in the 18th-century advances in probability, the modern field of statistics only emerged in the lateth and earlyth century in three stages. The first wave, at the turn of the century, was led by the work of Francis Galton and Karl Pearson , who transformed statistics into a rigorous mathematical discipline used for analysis, not just in science, but in industry and politics as well. The second wave of the s and 20s was initiated by William Gosset , and reached its culmination in the insights of Ronald Fisher. This involved the development of better design of experiments models, hypothesis testing and techniques for use with small data samples. The final wave, which mainly saw the refinement and expansion of earlier developments, emerged from the collaborative work between Egon Pearson and Jerzy Neyman in the s. The original logo of the Royal Statistical Society , founded in The first statistical bodies were established in the early 19th century. The Royal Statistical Society was founded in and Florence Nightingale , its first female member, pioneered the application of statistical analysis to health problems for the furtherance of epidemiological understanding and public health practice. However, the methods then used would not be considered as modern statistics today. His contributions to the field included introducing the concepts of standard deviation , correlation , regression and the application of these methods to the study of the variety of human characteristics - height, weight,

eyelash length among others. He found that many of these could be fitted to a normal curve distribution. The actual weight was pounds: The guesses were markedly non-normally distributed. Karl Pearson, the founder of mathematical statistics. His work grew to encompass the fields of biology, epidemiology, anthropometry, medicine and social history. In 1894, with Walter Weldon, founder of biometry, and Galton, he founded the journal *Biometrika* as the first journal of mathematical statistics and biometry. Ronald Fisher, "A genius who almost single-handedly created the foundations for modern statistical science", [34] The second wave of mathematical statistics was pioneered by Ronald Fisher who wrote two textbooks, *Statistical Methods for Research Workers*, published in 1930 and *The Design of Experiments* in 1935, that were to define the academic discipline in universities around the world. He also systematized previous results, putting them on a firm mathematical footing. In his seminal paper *The Correlation between Relatives on the Supposition of Mendelian Inheritance*, the first use to use the statistical term, variance. In 1906, at Rothamsted Experimental Station he started a major study of the extensive collections of data recorded over many years. This resulted in a series of reports under the general title *Studies in Crop Variation*. In 1918 he published *The Genetical Theory of Natural Selection* where he applied statistics to evolution. Over the next seven years, he pioneered the principles of the design of experiments see below and elaborated his studies of analysis of variance. He furthered his studies of the statistics of small samples. Perhaps even more important, he began his systematic approach of the analysis of real data as the springboard for the development of new statistical methods. He developed computational algorithms for analyzing data from his balanced experimental designs. In 1925, this work resulted in the publication of his first book, *Statistical Methods for Research Workers*. In 1935, this book was followed by *The Design of Experiments*, which was also widely used. In addition to analysis of variance, Fisher named and promoted the method of maximum likelihood estimation. Before this deviations exceeding three times the probable error were considered significant. For a symmetrical distribution the probable error is half the interquartile range. Jerzy Neyman in 1934 showed that stratified random sampling was in general a better method of estimation than purposive quota sampling. Please improve this section by adding secondary or tertiary sources. February Learn how and when to remove this template message James Lind carried out the first ever clinical trial in 1747, in an effort to find a treatment for scurvy. In 1747, while serving as surgeon on HM Bark *Salisbury*, James Lind carried out a controlled experiment to develop a cure for scurvy. The men were paired, which provided blocking. From a modern perspective, the main thing that is missing is randomized allocation of subjects to treatments. Lind is today often described as a one-factor-at-a-time experimenter. Peirce in "Illustrations of the Logic of Science" and "A Theory of Probable Inference", two publications that emphasized the importance of randomization-based inference in statistics. In another study, Peirce randomly assigned volunteers to a blinded, repeated-measures design to evaluate their ability to discriminate weights. He was described by Anders Hald as "a genius who almost single-handedly created the foundations for modern statistical science. Perhaps even more important, Fisher began his systematic approach to the analysis of real data as the springboard for the development of new statistical methods. He began to pay particular attention to the labour involved in the necessary computations performed by hand, and developed methods that were as practical as they were founded in rigour. In 1925, this work culminated in the publication of his first book, *Statistical Methods for Research Workers*. Fisher, in his innovative book *The Design of Experiments* which also became a standard. While this sounds like a frivolous application, it allowed him to illustrate the most important ideas of experimental design: Agricultural science advances served to meet the combination of larger city populations and fewer farms. But for crop scientists to take due account of widely differing geographical growing climates and needs, it was important to differentiate local growing conditions. To extrapolate experiments on local crops to a national scale, they had to extend crop sample testing economically to overall populations. As statistical methods advanced primarily the efficacy of designed experiments instead of one-factor-at-a-time experimentation, representative factorial design of experiments began to enable the meaningful extension, by inference, of experimental sampling results to the population as a whole. Bayesian statistics[ edit ] Pierre-Simon, marquis de Laplace, one of the main early developers of Bayesian statistics. However it was Pierre-Simon Laplace who introduced a general version of the theorem and applied it to celestial mechanics, medical statistics, reliability, and jurisprudence. After the 18th century, inverse probability was

largely supplanted[ citation needed ] by a collection of methods that were developed by Ronald A. Fisher , Jerzy Neyman and Egon Pearson. Their methods came to be called frequentist statistics. In the objectivist stream, the statistical analysis depends on only the model assumed and the data analysed. In contrast, "subjectivist" statisticians deny the possibility of fully objective analysis for the general case. His seminal book "Theory of probability" first appeared in and played an important role in the revival of the Bayesian view of probability. In the s, there was a dramatic growth in research and applications of Bayesian methods, mostly attributed to the discovery of Markov chain Monte Carlo methods, which removed many of the computational problems , and an increasing interest in nonstandard, complex applications.

**Chapter 5 : A History of Probability and Statistics and Their Applications Before by Anders Hald**

*Origin of Statistics and Probability* The original idea of "statistics" was the collection of information about and for the "state". The word statistics derives directly, not from any classical Greek or Latin roots, but from the Italian word for state.

Etymology[ edit ] Probable and probability and their cognates in other modern languages derive from medieval learned Latin probabilis and, deriving from Cicero and generally applied to an opinion to mean plausible or generally approved. In the 18th century, the term chance was also used in the mathematical sense of "probability" and probability theory was called Doctrine of Chances. This word is ultimately from Latin cadentia, i. The English adjective likely is of Germanic origin, most likely from Old Norse likligr Old English had geliclic with the same sense , originally meaning "having the appearance of being strong or able" "having the similar appearance or qualities", with a meaning of "probably" recorded from the late 14th century. Similarly, the derived noun likelihood had a meaning of "similarity, resemblance" but took on a meaning of "probability" from the mid 15th century. Timeline of probability and statistics Al-Kindi was the first to use statistics to decipher encrypted messages and developed the first code breaking algorithm in the House of Wisdom in Baghdad, based on frequency analysis. He wrote a book entitled "Manuscript on Deciphering Cryptographic Messages", containing detailed discussions on statistics. Christiaan Huygens gave a comprehensive treatment of the subject. In ancient times there were games played using astragali, or Talus bone. The Pottery of ancient Greece was evidence to show that there was a circle drawn on the floor and the astragali were tossed into this circle, much like playing marbles. In Egypt , excavators of tombs found a game they called "Hounds and Jackals", which closely resembles the modern game " Snakes and Ladders ". It seems that this is the early stages of the creation of dice. The first dice game mentioned in literature of the Christian era was called Hazard. Played with 2 or 3 dice. Thought to have been brought to Europe by the knights returning from the Crusades. Dante Alighieri mentions this game. A commentor of Dante puts further thought into this game: Achieving a 4 can be done with 3 die by having a two on one die and aces on the other two dice. Cardano also thought about the sum of three dice. At face value there are the same number of combinations that sum to 9 as those that sum to 10. However, there are more ways of obtaining some of these combinations than others. For example, if we consider the order of results there are six ways to obtain 10. There are a total of 27 permutations that sum to 10 but only 25 that sum to 9. From this, Cardano found that the probability of throwing a 9 is less than that of throwing a 10. He also demonstrated the efficacy of defining odds as the ratio of favourable to unfavourable outcomes which implies that the probability of an event is given by the ratio of favourable outcomes to the total number of possible outcomes [7]. In addition, Galileo wrote about die-throwing sometime between 1613 and 1623. Bernoulli proved a version of the fundamental law of large numbers , which states that in a large number of trials, the average of the outcomes is likely to be very close to the expected value - for example, in throws of a fair coin, it is likely that there are close to heads and the larger the number of throws, the closer to half-and-half the proportion is likely to be. The theory of errors used the method of least squares to correct error-prone observations, especially in astronomy, based on the assumption of a normal distribution of errors to determine the most likely true value. Towards the end of the nineteenth century, a major success of explanation in terms of probabilities was the Statistical mechanics of Ludwig Boltzmann and J. Willard Gibbs which explained properties of gases such as temperature in terms of the random motions of large numbers of particles. Twentieth century[ edit ] Probability and statistics became closely connected through the work on hypothesis testing of R. Fisher and Jerzy Neyman , which is now widely applied in biological and psychological experiments and in clinical trials of drugs, as well as in economics and elsewhere. A hypothesis, for example that a drug is usually effective, gives rise to a probability distribution that would be observed if the hypothesis is true. If observations approximately agree with the hypothesis, it is confirmed, if not, the hypothesis is rejected. That provided a model for the study of random fluctuations in stock markets, leading to the use of sophisticated probability models in mathematical finance , including such successes as the widely used Black-Scholes formula for the valuation of options. In the

mid-century frequentism was dominant, holding that probability means long-run relative frequency in a large number of trials. At the end of the century there was some revival of the Bayesian view, according to which the fundamental notion of probability is how well a proposition is supported by the evidence for it. Franklin, The Science of Conjecture: Evidence and Probability Before Pascal, , References[ edit ] Bernstein, Peter L. The Remarkable Story of Risk. Classical Probability in the Enlightenment. The Science of Conjecture: Evidence and Probability Before Pascal. Johns Hopkins University Press. The Emergence of Probability 2nd ed. A History of Mathematical Statistics from to Statisticians of the Centuries. The Lady Tasting Tea:

**Chapter 6 : Probability Symbols, Names, and Explanations Every Statistician Should Know**

• *History of Statistics and Probability 18 short biographies from University of Minnesota Morris.* • *Glimpses of the Prehistory of Econometrics. Montage by Jan Kiviet.* • *Probability and Statistics Ideas in the Classroom: Lesson from History. Comments on the uses of history by D. R. Bellhouse.*

Meaning, Concept and Importance Statistics Article shared by: After reading this article you will learn about: Meaning of Probability 2. Different Schools of Thought on the Concept of Probability 3. All these terms, possibility and probability convey the same meaning. The theory of probability has been developed in 17th century. It has got its origin from games, tossing coins, throwing a dice, drawing a card from a pack. In Antoine Gornband had taken an initiation and an interest for this area. After him many authors in statistics had tried to remodel the idea given by the former. Sometimes statistical analysis becomes paralyzed without the theorem of probability. The probability is zero for an impossible event and one for an event which is certain to occur. The probability that the sky will fall is. An individual now living will some day die is 1. Let us clarify the meaning of probability with an example of drawing a playing card. The probability of an event stated or expressed mathematically called as a ratio. These ratios, called probability ratios, are defined by that fraction, the numerator of which equals the desired outcome or outcomes, and the denominator of which equals the total possible outcomes. More simply put, the probability of the appearance of any face on a 6-faced e. Zero for an event which cannot occur and 1 for an event, certain to occur. Different Schools of Thought on the Concept of Probability: There are different schools of thought on the concept of probability: The classical approach to probability is one of the oldest and simplest school of thought. It has been originated in 18th century which explains probability concerning games of chances such as throwing coin, dice, drawing cards etc. According to him probability is the ratio of the number of favourable cases among the number of equally likely cases. Or in other words, the ratio suggested by classical approach is: When probability is zero it denotes that it is impossible to occur. If probability is 1 then there is certainty for occurrence, i. From a bag containing 20 black and 25 white balls, a ball is drawn randomly. What is the probability that it is black. Relative Frequency Theory of Probability: This approach to probability is a protest against the classical approach. If there are two types of objects among the objects of similar or other natures then the probability of one object i. This approach is not at all an authentic and scientific approach. This approach of probability is an undefined concept. This type of probability approach though applied in business and economics area still then it is not a reliable one. Important Terminology in Probability: The events are said to be mutually exclusive when they are not occurred simultaneously. Among the events, if one event will remain present in a trial other events will not appear. In other words, occurrence of one precludes the occurrence of all the others. If a girl is beautiful, she cannot be ugly. If a ball is white, it cannot be red. If we take another events like dead and alive, it can be said that a person may be either alive or dead at a point of time. But lie cannot be both alive and dead simultaneously. If a coin is tossed either the head will appear or tail will appear. But both cannot appear in the same time. It refers that in tossing a coin the occurrence of head and tail comes under mutually exclusive events. Independent and Dependent Events: Two or more events are said to be independent when the occurrence of one trial does not affect the other. It indicates the fact that if trial made one by one, one trial is not affected by the other trial. And also one trial never describes anything about the other trials. The events in tossing a coin are independent events. If a coin is tossed one by one, then one trial is not affected by the other. In a trial the head or tail may conic which never describes anything what event will come in second trial. So the second trial is completely independent to that of the first trial. Dependent events are those in which the occurrence and non-occurrence of one event in a trial may affect the occurrence of the other trials. Here the events are mutually dependent on each other. If a card is drawn from a pack of playing cards and is not replaced, then in 2nd trial probability will be altered. Events are said to be equally likely, when there is equal chance of occurring. If one event is not occurred like other events then events are not considered as equally likely. Or in other words events are said to be equally likely when one event does not occur more often than the others. If an unbiased coin or dice is thrown, each face may be expected to occur is equal numbers in the

long run. In other example, in a pack of playing cards we expect each card to appear equally. If a coin or dice is biased then each face is not expected to appear equally. Simple and Compound Events: In the simple events we think about the probability of the happening or not-happening of the simple events. Whenever we are tossing the coin we are considering the occurrence of the events of head and tail. In another example, if in a bag there are 10 white balls and 6 red balls and whenever we are trying to find out the probability of drawing a red ball, is included in simple events. But on the other hand when we consider the joint occurrence of two or more events, it becomes compound events. Unlike simple events here more than one event are taken into consideration. If there are 10 white and 6 red balls in a bag and if successive draws of 3 balls are made and when we are trying to find out the probability of 3 balls as the white balls. This example states the fact that the events are considered in more than two eventual cases. The concept of probability is of great importance in everyday life. Statistical analysis is based on this valuable concept. In fact the role played by probability in modern science is that of a substitute for certainty. The following discussion explains it further: The probability theory is very much helpful for making prediction. Estimates and predictions form an important part of research investigation. With the help of statistical methods, we make estimates for the further analysis. Thus, statistical methods are largely dependent on the theory of probability. It has also immense importance in decision making. It is concerned with the planning and controlling and with the occurrence of accidents of all kinds. It is one of the inseparable tools for all types of formal studies that involve uncertainty. The concept of probability is not only applied in business and commercial lines, rather than it is also applied to all scientific investigation and everyday life. Before knowing statistical decision procedures one must have to know about the theory of probability. The characteristics of the Normal Probability. Curve is based upon the theory of probability. Normal Distribution is by far the most used distribution for drawing inferences from statistical data because of the following reasons: Number of evidences are accumulated to show that normal distribution provides a good fit or describe the frequencies of occurrence of many variables and facts in i biological statistics e. Normal distribution is of great value in evaluation and research in both psychology and education, when we make use of mental measurement. It may be noted that normal distribution is not an actual distribution of scores on any test of ability or academic achievement, but is instead, a mathematical model. The distribution of test scores approach the theoretical normal distribution as a limit, but the fit is rarely ideal and perfect. Principles of Probability and Normal Probability Curve: When we toss an unbiased coin it may fall head or tail. If we toss two unbiased coins, they may fall in a number of ways as HH two heads HT 1st coin head and 2nd coin tail , TH 1st coin-tail and 2nd coin head or TT two tails. So there are four possible arrangements if we toss two coins, a and b , at the same time: The expansion has eleven combinations and the chance of occurrence of each combination out of the total possible occurrence is expressed by the coefficient of each combination. We can represent the above eleven terms of the expansion along X-axis at equal distances as: We can represent the chance of occurrence of each combination of H and T as frequencies along Y axis. If we plot all these points and join them we shall get a symmetrical frequency polygon. Carefully look at the following frequency distribution, which a teacher obtained after examining students of class IX on a mathematics achievement test see Table 6.

**Chapter 7 : Origin of Statistics and Probability: Where, When, By Whom, How**

*History of Statistics. and analysis of data. and analytical work which requires statistical inference. the term "statistics" designated the systematic collection of demographic and economic data by states.*

Fitting equations to data prior to Many, but not all, of the works included here are mentioned above. This collects together all identified by Merriman in his list of writings on least squares up to, but not including Legendre. Volume 1 lists pure mathematical papers. Those in probability Classification are further subdivided into the Theory of Errors including Least Squares Method and Probabilities including Problems. Statistics Classification includes Actuarial Mathematics with subdivisions of Mortality and Population. The lists of publications has been generated from the Catalog of Scientific Papers and the lists constructed by Merriman and Harter for publications related to least squares. There is a large actuarial literature for the most part not included. Note that the date of reading and the date of publication are frequently many years apart. This period is marked especially by the efforts devoted to the theory of errors and the method of least squares. Manfield Merriman in his A List of writings relating to the Method of Least Squares has supplied a list of titles related to it carried from to Comments on particular papers are taken from Merriman. The thirteen "proofs" of the Method of Least Squares: Several of those who are more important than others are given separate listings of papers. Namely, Adrien-Marie Legendre who first published in on the method of least squares. His works in this area span to Although Legendre was the first to publish, he claimed to have used the procedure twelve years before Legendre published. He also investigated the optimal size of juries and the number of jurors required to convict. His work spans to See also Cauchy below for early papers. We include here a series of papers on determining the orbits of planets and asteroids. George Boole who was a logician. Adolphe Quetelet who worked in the social sciences.

**Chapter 8 : History of Statistics & Probability**

*A Short History of Probability From Calculus, Volume II by Tom M. Apostol (2 nd edition, John Wiley & Sons, ): "A gambler's dispute in led to the creation of a mathematical theory of probability by two famous French mathematicians, Blaise Pascal and Pierre de Fermat.*

Early probability Games of chance The modern mathematics of chance is usually dated to a correspondence between the French mathematicians Pierre de Fermat and Blaise Pascal in 1654. Suppose two players, A and B, are playing a three-point game, each having wagered 32 pistoles, and are interrupted after A has two points and B has one. How much should each receive? Fermat and Pascal proposed somewhat different solutions, though they agreed about the numerical answer. Each undertook to define a set of equal or symmetrical cases, then to answer the problem by comparing the number for A with that for B. Fermat, however, gave his answer in terms of the chances, or probabilities. He reasoned that two more games would suffice in any case to determine a victory. There are four possible outcomes, each equally likely in a fair game of chance. Of these four sequences, only the last would result in a victory for B. Thus, the odds for A are 3:1. In that case, the positions of A and B would be equal, each having won two games, and each would be entitled to 32 pistoles. A should receive his portion in any case. This first round can now be treated as a fair game for this stake of 32 pistoles, so that each player has an expectation of 16 pistoles. Games of chance such as this one provided model problems for the theory of chances during its early period, and indeed they remain staples of the textbooks. Fermat and Pascal were not the first to give mathematical solutions to problems such as these. More than a century earlier, the Italian mathematician, physician, and gambler Girolamo Cardano calculated odds for games of luck by counting up equally probable cases. His little book, however, was not published until 1564, by which time the elements of the theory of chances were already well known to mathematicians in Europe. It will never be known what would have happened had Cardano published in the 16th century. It cannot be assumed that probability theory would have taken off in the 16th century. Cardano, moreover, had no great faith in his own calculations of gambling odds, since he believed also in luck, particularly in his own. In the Renaissance world of monstrosities, marvels, and similitudes, chance—“allied to fate”—was not readily naturalized, and sober calculation had its limits. It was, for example, used by the Dutch mathematician Christiaan Huygens in his short treatise on games of chance, published in 1657. Huygens refused to define equality of chances as a fundamental presumption of a fair game but derived it instead from what he saw as a more basic notion of an equal exchange. Most questions of probability in the 17th century were solved, as Pascal solved his, by redefining the problem in terms of a series of games in which all players have equal expectations. The new theory of chances was not, in fact, simply about gambling but also about the legal notion of a fair contract. A fair contract implied equality of expectations, which served as the fundamental notion in these calculations. Measures of chance or probability were derived secondarily from these expectations. Probability was tied up with questions of law and exchange in one other crucial respect. Chance and risk, in aleatory contracts, provided a justification for lending at interest, and hence a way of avoiding Christian prohibitions against usury. Lenders, the argument went, were like investors; having shared the risk, they deserved also to share in the gain. For this reason, ideas of chance had already been incorporated in a loose, largely nonmathematical way into theories of banking and marine insurance. From about 1650, initially in the Netherlands, probability began to be used to determine the proper rates at which to sell annuities. Jan de Wit, leader of the Netherlands from 1654 to 1672, corresponded in the 1650s with Huygens, and eventually he published a small treatise on the subject of annuities in 1671. Annuities in early modern Europe were often issued by states to raise money, especially in times of war. This formula took no account of age at the time the annuity was purchased. Wit lacked data on mortality rates at different ages, but he understood that the proper charge for an annuity depended on the number of years that the purchaser could be expected to live and on the presumed rate of interest. Despite his efforts and those of other mathematicians, it remained rare even in the 18th century for rulers to pay much heed to such quantitative considerations. Life insurance, too, was connected only loosely to probability calculations and mortality records, though statistical data on death became increasingly available in the course

of the 18th century. The first insurance society to price its policies on the basis of probability calculations was the Equitable, founded in London in 1762. But from medieval times to the 18th century and even into the 19th, a probable belief was most often merely one that seemed plausible, came on good authority, or was worthy of approval. Probability, in this sense, was emphasized in England and France from the late 17th century as an answer to skepticism. Man may not be able to attain perfect knowledge but can know enough to make decisions about the problems of daily life. The new experimental natural philosophy of the later 17th century was associated with this more modest ambition, one that did not insist on logical proof. Almost from the beginning, however, the new mathematics of chance was invoked to suggest that decisions could after all be made more rigorous. Perhaps, he supposed, the unbeliever can be persuaded by consideration of self-interest. If there is a God Pascal assumed he must be the Christian God, then to believe in him offers the prospect of an infinite reward for infinite time. However small the probability, provided only that it be finite, the mathematical expectation of this wager is infinite. It seemed plain which was the more reasonable choice. The link between the doctrine of chance and religion remained an important one through much of the 18th century, especially in Britain. Another argument for belief in God relied on a probabilistic natural theology. The classic instance is a paper read by John Arbuthnot to the Royal Society of London in 1710 and published in its *Philosophical Transactions* in 1713. Arbuthnot presented there a table of christenings in London from 1686 to 1710. He observed that in every year there was a slight excess of male over female births. The proportion, approximately 14 boys for every 13 girls, was perfectly calculated, given the greater dangers to which young men are exposed in their search for food, to bring the sexes to an equality of numbers at the age of marriage. Could this excellent result have been produced by chance alone? Arbuthnot thought not, and he deployed a probability calculation to demonstrate the point. The probability that male births would by accident exceed female ones in 82 consecutive years is  $0$ . Considering further that this excess is found all over the world, he said, and within fixed limits of variation, the chance becomes almost infinitely small. This argument for the overwhelming probability of Divine Providence was repeated by many and refined by a few. Nicolas Bernoulli, from the famous Swiss mathematical family, gave a more skeptical view. If the underlying probability of a male birth was assumed to be  $0$ . That is, no Providential direction was required. Apart from natural theology, probability came to be seen during the 18th-century Enlightenment as a mathematical version of sound reasoning. In the German mathematician Gottfried Wilhelm Leibniz imagined a utopian world in which disagreements would be met by this challenge: For there were some cases where a straightforward application of probability mathematics led to results that seemed to defy rationality. One example, proposed by Nicolas Bernoulli and made famous as the St. Petersburg paradox, involved a bet with an exponentially increasing payoff. A fair coin is to be tossed until the first time it comes up heads. If it comes up heads on the first toss, the payment is 2 ducats; if the first time it comes up heads is on the second toss, 4 ducats; and if on the  $n$ th toss,  $2n$  ducats. The mathematical expectation of this game is infinite, but no sensible person would pay a very large sum for the privilege of receiving the payoff from it. The disaccord between calculation and reasonableness created a problem, addressed by generations of mathematicians. Smallpox was at this time widespread and deadly, infecting most and carrying off perhaps one in seven Europeans. Inoculation in these days involved the actual transmission of smallpox, not the cowpox vaccines developed in the 18th century by the English surgeon Edward Jenner, and was itself moderately risky. Was it rational to accept a small probability of an almost immediate death to reduce greatly a large probability of death by smallpox in the indefinite future? Calculations of mathematical expectation, as by Daniel Bernoulli, led unambiguously to a favourable answer. One might, he argued, reasonably prefer a greater assurance of surviving in the near term to improved prospects late in life. Jakob Bernoulli, uncle of Nicolas and Daniel, formulated and proved a law of large numbers to give formal structure to such reasoning. This was published in 1713 from a manuscript, the *Ars conjectandi*, left behind at his death in 1705. There he showed that the observed proportion of, say, tosses of heads or of male births will converge as the number of trials increases to the true probability  $p$ , supposing that it is uniform. His theorem was designed to give assurance that when  $p$  is not known in advance, it can properly be inferred by someone with sufficient experience. He thought of disease and the weather as in some way like drawings from an urn. At bottom they are deterministic, but since one cannot know the causes in sufficient

detail, one must be content to investigate the probabilities of events under specified conditions. But Hartley named no names, and the first publication of the formula he promised occurred in a posthumous paper of Thomas Bayes, communicated to the Royal Society by the British philosopher Richard Price. The result was perhaps more consequential in theory than in practice. Laplace and his more politically engaged fellow mathematicians, most notably Marie-Jean-Antoine-Nicolas de Caritat, marquis de Condorcet, hoped to make probability into the foundation of the moral sciences. This took the form principally of judicial and electoral probabilities, addressing thereby some of the central concerns of the Enlightenment philosophers and critics. Justice and elections were, for the French mathematicians, formally similar. In each, a crucial question was how to raise the probability that a jury or an electorate would decide correctly. One element involved testimonies, a classic topic of probability theory. In the British mathematician John Craig used probability to vindicate the truth of scripture and, more idiosyncratically, to forecast the end of time, when, due to the gradual attrition of truth through successive testimonies, the Christian religion would become no longer probable. The Scottish philosopher David Hume, more skeptically, argued in probabilistic but nonmathematical language beginning in that the testimonies supporting miracles were automatically suspect, deriving as they generally did from uneducated persons, lovers of the marvelous. Miracles, moreover, being violations of laws of nature, had such a low a priori probability that even excellent testimony could not make them probable. Condorcet also wrote on the probability of miracles, or at least faits extraordinaires, to the end of subduing the irrational. But he took a more sustained interest in testimonies at trials, proposing to weigh the credibility of the statements of any particular witness by considering the proportion of times that he had told the truth in the past, and then use inverse probabilities to combine the testimonies of several witnesses. Laplace and Condorcet applied probability also to judgments. In contrast to English juries, French juries voted whether to convict or acquit without formal deliberations. There would be no injustice, Condorcet argued, in exposing innocent defendants to a risk of conviction equal to risks they voluntarily assume without fear, such as crossing the English Channel from Dover to Calais. Using this number and considering also the interest of the state in minimizing the number of guilty who go free, it was possible to calculate an optimal jury size and the majority required to convict. But by this time the whole enterprise had come to seem gravely doubtful, in France and elsewhere.

## Chapter 9 : A Short History of Probability

*From the Reviews of History of Probability and Statistics and Their Applications before "This is a marvelous book Anyone with the slightest interest in the history of statistics, or in understanding how modern ideas have developed, will find this an invaluable resource."*